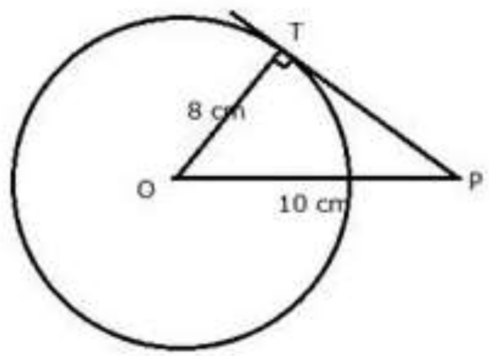
Tangents and Intersecting Chords

Exercise 18A

Question 1.
The radius of a circle is 8 cm. Calculate the length of a tangent drawn to this circle from a point at a distance of 10 cm from its centre?

Solution:



OP = 10 cm; radius OT = 8 cm

$\because OT \perp PT$

In Rt. ΔOTP,

$OP^2 = OT^2 + PT^2$

$10^2 = 8^2 + PT^2$

$PT^2 = 100 - 64$

$PT^2 = 36$

$PT = 6$

Length of tangent = 6 cm.

Question 2.

In the given figure, O is the centre of the circle and AB is a tangent to the circle at B. If AB = 15 cm and AC = 7.5 cm, calculate the radius of the circle.

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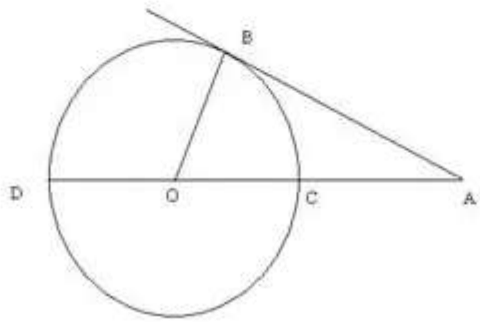


Solution:

AB = 15 cm, AC = 7.5 cm

Let 'r' be the radius of the circle.

$\therefore$ OC = OB = r

AO = AC + OC = 7.5 + r

In ΔAOB,

$AO^2 = AB^2 + OB^2$

$(7.5 + r)^2 = 15^2 + r^2$

$\Rightarrow \left(\frac{15 + 2r}{2}\right)^2 = 225 + r^2$

$\Rightarrow 225 + 4r^2 + 60r = 900 + 4r^2$

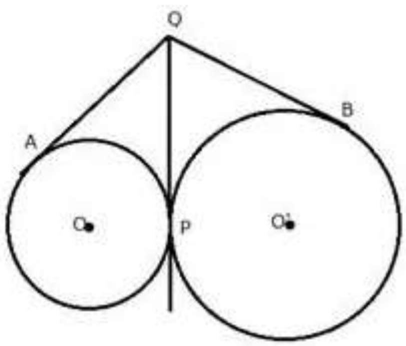
$\Rightarrow 60r = 675$

$\Rightarrow r = 11.25$ cm

Therefore, r = 11.25 cm

Question 3.

Two circles touch each other externally at point P. Q is a point on the common tangent through P. Prove that the tangents QA and QB are equal.



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Solution:

From Q, QA and QP are two tangents to the circle with centre O

Therefore, QA = QP .....(i)

Similarly, from Q, QB and QP are two tangents to the circle with centre O'

Therefore, QB = QP ......(ii)

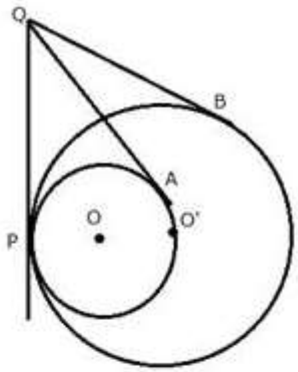
From (i) and (ii)

QA = QB

Therefore, tangents QA and QB are equal.

Question 4.
Two circles touch each other internally. Show that the tangents drawn to the two circles from any point on the common tangent are equal in length.

Solution:



From Q, QA and QP are two tangents to the circle with centre O
Therefore, QA = QP ....(i)
Similarly, from Q, QB and QP are two tangents to the circle with centre O'
Therefore, QB = QP ....(ii)
From (i) and (ii)
QA = QB
Therefore, tangents QA and QB are equal.

Question 5.
Two circles of radii 5 cm and 3 cm are concentric. Calculate the length of a chord of the outer circle which touches the inner.

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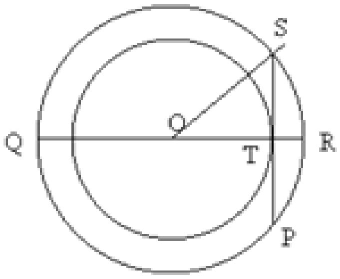
Solution:



OS = 5 cm

OT = 3 cm

In Rt. Triangle OST

By Pythagoras Theorem,

$$ST^2 = OS^2 - OT^2$$
$$ST^2 = 25 - 9$$
$$ST^2 = 16$$
$$ST = 4 \text{ cm}$$

Since OT is perpendicular to SP and OT bisects chord SP

So, SP = 8 cm

Question 6.

Three circles touch each other externally. A triangle is formed when the centers of these circles are joined together. Find the radii of the circles, if the sides of the triangle formed are 6 cm, 8 cm and 9 cm.

Solution:

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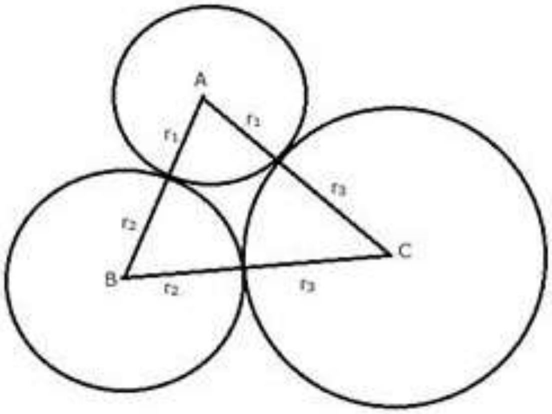


AB = 6 cm, AC = 8 cm and BC = 9 cm

Let radii of the circles having centers A, B and C be r_1, r_2 and r_3 respectively.

$r_1 + r_3 = 8$

$r_3 + r_2 = 9$

$r_2 + r_1 = 6$

Adding

$r_1 + r_3 + r_3 + r_2 + r_2 + r_1 = 8+9+6$

$2(r_1 + r_2 + r_3) = 23$

$r_1 + r_2 + r_3 = 11.5$ cm

$r_1 + 9 = 11.5$ (Since $r_2 + r_3 = 9$)

$r_1 = 2.5$ cm

$r_2 + 6 = 11.5$ (Since $r_1 + r_3 = 6$)

$r_2 = 5.5$ cm

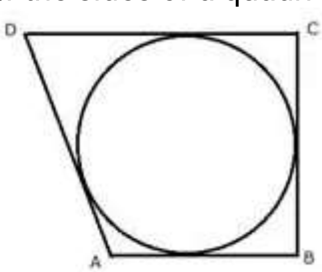
$r_3 + 8 = 11.5$ (Since $r_2 + r_1 = 8$)

$r_3 = 3.5$ cm

Hence, $r_1 = 2.5$ cm, $r_2 = 5.5$ cm and $r_3 = 3.5$ cm

Question 7.

If the sides of a quadrilateral ABCD touch a circle, prove that AB + CD = BC + AD.



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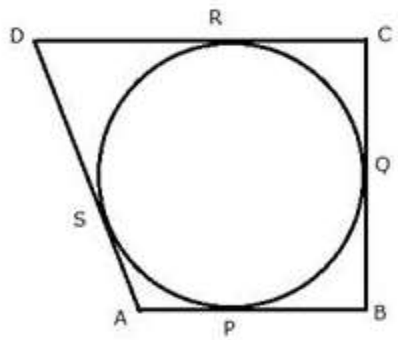
Solution:



Let the circle touch the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively.

Since AP and AS are tangents to the circle from external point A

AP = AS ....(i)

Similarly, we can prove that:

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

Adding,

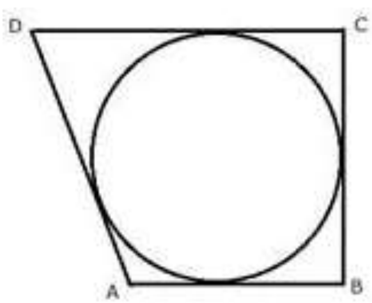
AP + BP + CR + DR = AS + DS + BQ + CQ

AB + CD = AD + BC

Hence, AB + CD = AD + BC

Question 8.

If the sides of a parallelogram touch a circle, prove that the parallelogram is a rhombus.



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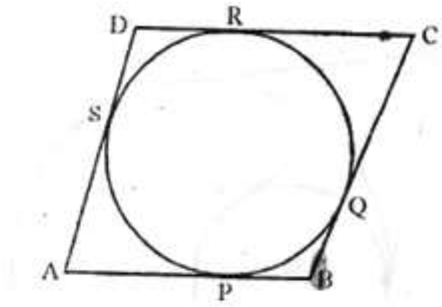
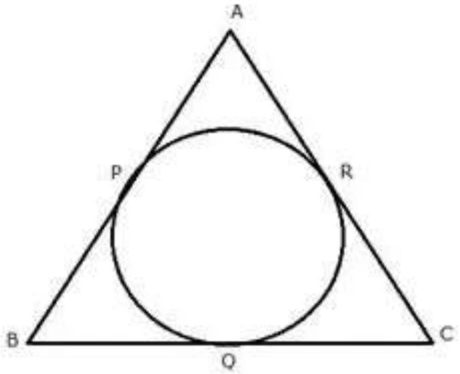
Solution:



From A, AP and AS are tangents to the circle.
Therefore, AP = AS.......(i)

Similarly, we can prove that:
BP = BQ ....(ii)
CR = CQ ....(iii)
DR = DS ....(iv)
Adding,
AP + BP + CR + DR = AS + DS + BQ + CQ
AB + CD = AD + BC
Hence, AB + CD = AD + BC
But AB = CD and BC = AD.......(v) Opposite sides of a ||gm
Therefore, AB + AB = BC + BC
2AB = 2 BC
AB = BC ....(vi)
From (v) and (vi)
AB = BC = CD = DA
Hence, ABCD is a rhombus.

Question 9.
From the given figure prove that:
AP + BQ + CR = BP + CQ + AR.



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Also, show that AP + BQ + CR = $\frac{1}{2}$ × perimeter of triangle ABC.

Solution:

Since from B, BQ and BP are the tangents to the circle

Therefore, BQ = BP ....(i)

Similarly, we can prove that

AP = AR ....(ii)

and CR = CQ ....(iii)

Adding,

AP + BQ + CR = BP + CQ + AR ....(iv)

Adding AP + BQ + CR to both sides

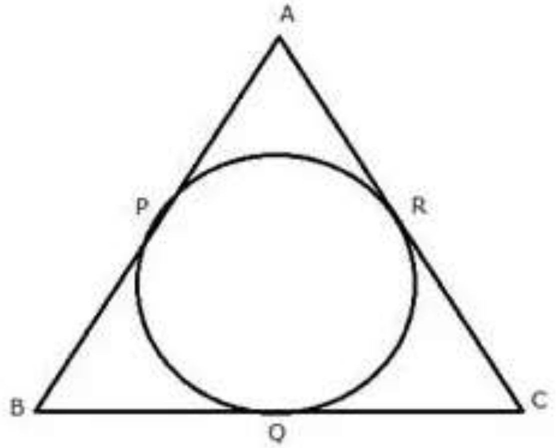
2(AP + BQ + CR) = AP + PQ + CQ + QB + AR + CR

2(AP + BQ + CR) = AB + BC + CA

Therefore, AP + BQ + CR = $\frac{1}{2}$ x (AB + BC + CA)

AP + BQ + CR = $\frac{1}{2}$ x perimeter of triangle ABC

Question 10.
In the figure, if AB = AC then prove that BQ = CQ.



Solution:

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Since, from A, AP and AR are the tangents to the circle

Therefore, AP = AR

Similarly, we can prove that

BP = BQ and CR = CQ

Adding,

AP + BP + CQ = AR + BQ + CR

(AP + BP) + CQ = (AR + CR) + BQ

AB + CQ = AC + BQ

But AB = AC

Therefore, CQ = BQ or BQ = CQ

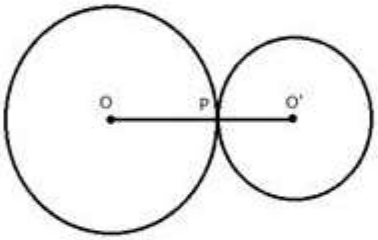
Question 11.
Radii of two circles are 6.3 cm and 3.6 cm. State the distance between their centers if –
i) they touch each other externally.
ii) they touch each other internally.

Solution:

Radius of bigger circle = 6.3 cm

and of smaller circle = 3.6 cm

i)



Two circles are touching each other at P externally. O and O' are the centers of the circles. Join OP and O'P

OP = 6.3 cm, O'P = 3.6 cm

Adding,

OP + O'P = 6.3 + 3.6 = 9.9 cm

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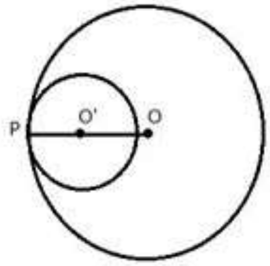
ii)



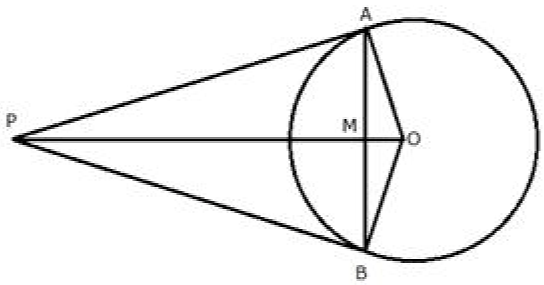
Two circles are touching each other at P internally. O and O' are the centers of the circles. Join OP and O'P

OP = 6.3 cm, O'P = 3.6 cm

OO' = OP - O'P = 6.3 - 3.6 = 2.7 cm

Question 12.
From a point P outside the circle, with centre O, tangents PA and PB are drawn. Prove that:
i) ∠AOP = ∠BOP
ii) OP is the perpendicular bisector of chord AB.

Solution:



i) In ΔAOP and ΔBOP

AP = BP (Tangents from P to the circle)

OP = OP (Common)

OA = OB (Radii of the same circle)

$\therefore$ By Side - Side - Side criterion of congruence,
$\Delta AOP \cong \Delta BOP$
The corresponding parts of the congruent triangles are congruent.
$\Rightarrow \angle AOP = \angle BOP$ [by c.p.c.t]

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ii) In ΔOAM and ΔOBM

OA = OB (Radii of the same circle)

$\angle AOM = \angle BOM$ (Proved $\angle AOP = \angle BOP$)

OM = OM (Common)

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta OAM \cong \Delta OBM$

The corresponding parts of the congruent triangles are congruent.

$\Rightarrow AM = MB$

and $\angle OMA = \angle OMB$

but,

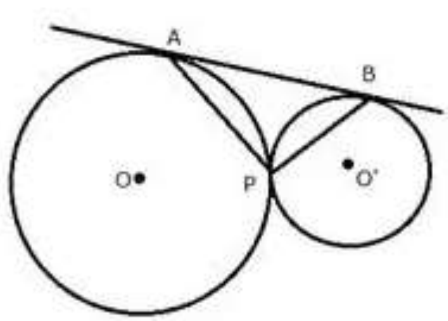
$\angle OMA + \angle OMB = 180^\circ$

$\therefore \angle OMA = \angle OMB = 90^\circ$

Hence, OM or OP is the perpendicular bisector of chord AB.

Question 13.

In the given figure, two circles touch each other externally at point P. AB is the direct common tangent of these circles. Prove that:



i) tangent at point P bisects AB.
ii) Angle APB = 90°

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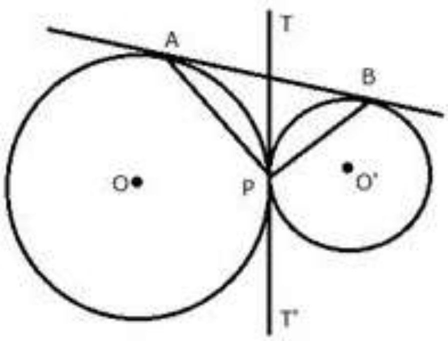
Solution:



Draw TPT' as common tangent to the circles.

i) TA and TP are the tangents to the circle with centre O.

Therefore, TA = TP ....(i)

Similarly, TP = TB ....(ii)

From (i) and (ii)

TA = TB

Therefore, TPT' is the bisector of AB.

ii) Now in ΔATP,

$\therefore \angle TAP = \angle TPA$

Similarly in ΔBTP, $\angle TBP = \angle TPB$

Adding,

$\angle TAP + \angle TBP = \angle APB$

But

$\therefore \angle TAP + \angle TBP + \angle APB = 180°$

$\Rightarrow \angle APB = \angle TAP + \angle TBP = 90°$

Question 14.

Tangents AP and AQ are drawn to a circle, with centre O, from an exterior point A. Prove that:

$\angle PAQ = 2\angle OPQ$

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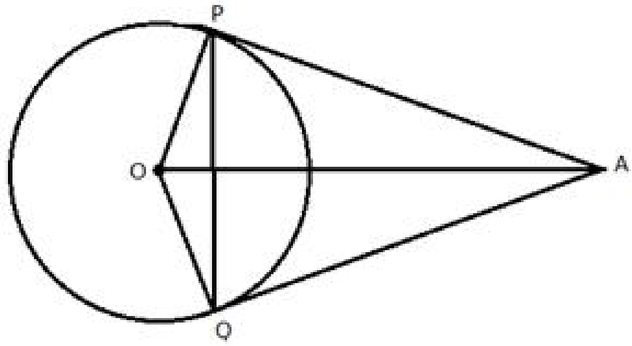
Solution:



In quadrilateral OPAQ,

$\angle OPA = \angle OQA = 90^\circ$
$(\because OP \perp PA \text{ and } OQ \perp QA)$
$\therefore \angle POQ + \angle PAQ + 90^\circ + 90^\circ = 360^\circ$
$\Rightarrow \angle POQ + \angle PAQ = 360^\circ - 180^\circ = 180^\circ$..........(i)

In triangle OPQ,

OP = OQ (Radii of the same circle)

$\therefore \angle OPQ = \angle OQP$
But
$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$
$\Rightarrow \angle POQ + \angle OPQ + \angle OPQ = 180^\circ$
$\Rightarrow \angle POQ + 2\angle OPQ = 180^\circ$............(ii)

From (i) and (ii)

$\angle POQ + \angle PAQ = \angle POQ + 2\angle OPQ$
$\Rightarrow \angle PAQ = 2\angle OPQ$

Question 15.
Two parallel tangents of a circle meet a third tangent at point P and Q. Prove that PQ subtends a right angle at the centre.

Solution:

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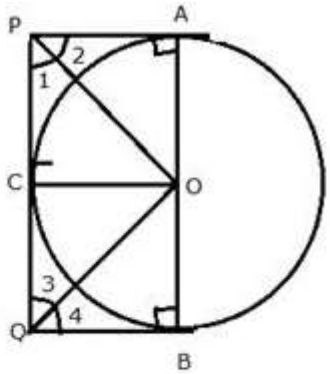

Join OP, OQ, OA, OB and OC.

In ΔOAP and ΔOCP

OA = OC (Radii of the same circle)

OP = OP (Common)

PA = PC (Tangents from P)

$\therefore$ By Side-Side-Side criterion of congruence,

$\Delta OAP \cong \Delta OCP$ (SSS Postulate)

The corresponding parts of the congruent trinagles are congruent.

$\Rightarrow \angle APO = \angle CPO$ (cpct)........(i)

Similarly, we can prove that

$\therefore \Delta OCQ \cong \Delta OBQ$

$\Rightarrow \angle CQO = \angle BQO$........(ii)

$\therefore \angle APC = 2\angle CPO$ and $\angle CQB = 2\angle CQO$

But,

$\angle APC + \angle CQB = 180^\circ$

(Sum of interior angles of a transversal)

$\therefore 2\angle CPO + 2\angle CQO = 180^\circ$

$\Rightarrow \angle CPO + \angle CQO = 90^\circ$

Now in ΔPOQ,

$\angle CPO + \angle CQO + \angle POQ = 180^\circ$

$\Rightarrow 90^\circ + \angle POQ = 180^\circ$

$\therefore \angle POQ = 90^\circ$

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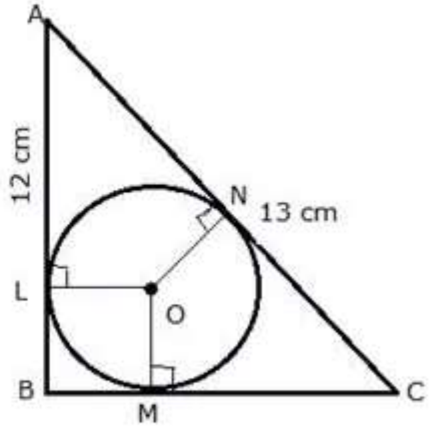
Question 16.
ABC is a right angled triangle with AB = 12 cm and AC = 13 cm. A circle, with centre O, has been inscribed inside the triangle.



Calculate the value of x, the radius of the inscribed circle.

Solution:

In $\Delta ABC, \angle B = 90°$

$OL \perp AB, OM \perp BC$ and $ON \perp AC$

LBNO is a square.

LB = BN = OL = OM = ON = x

$\therefore AL = 12 - x$
$\therefore AL = AN = 12 - x$

Since ABC is a right triangle

$AC^2 = AB^2 + BC^2$
$\Rightarrow 13^2 = 12^2 + BC^2$
$\Rightarrow 169 = 144 + BC^2$
$\Rightarrow BC^2 = 25$
$\Rightarrow BC = 5$
$\therefore MC = 5 - x$
But $CM = CN$
$\therefore CN = 5 - x$
Now, $AC = AN + NC$
$13 = (12 - x) + (5 - x)$
$13 = 17 - 2x$
$2x = 4$
$x = 2$ cm

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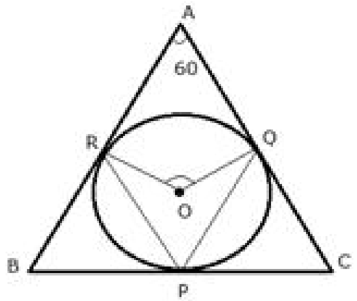
Question 17.
In a triangle ABC, the in circle (centre O) touches BC, CA and AB at points P, Q and R respectively. Calculate:
i) ∠QOR
ii) ∠QPR
given that ∠A = 60°

Solution:



The incircle touches the sides of the triangle ABC and

$OP \perp BC, OQ \perp AC, OR \perp AB$

i) In quadrilateral AROQ,

$\angle ORA = 90^\circ, \angle OQA = 90^\circ, \angle A = 60^\circ$
$\angle QOR = 360^\circ - (90^\circ + 90^\circ + 60^\circ)$
$\angle QOR = 360^\circ - 240^\circ$
$\angle QOR = 120^\circ$

ii) Now arc RQ subtends $\angle QOR$ at the centre and $\angle QPR$ at the remaining part of the circle.

$\therefore \angle QPR = \frac{1}{2}\angle QOR$

$\Rightarrow \angle QPR = \frac{1}{2} \times 120^\circ$

$\Rightarrow \angle QPR = 60^\circ$

Question 18.
In the following figure, PQ and PR are tangents to the circle, with centre O. If, calculate:
i) ∠QOR
ii) ∠OQR
iii) ∠QSR

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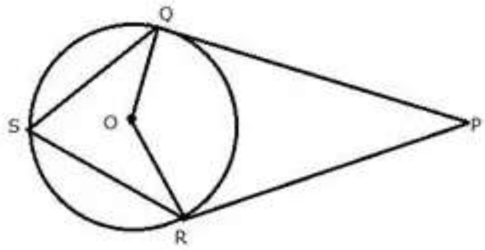
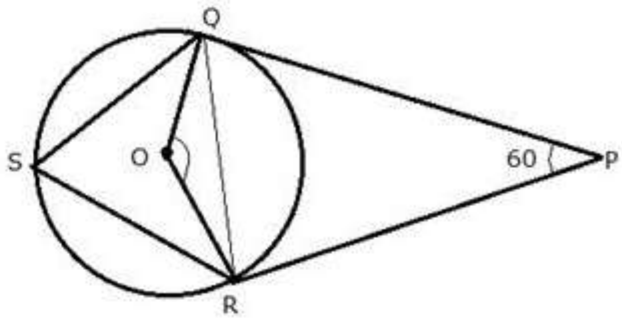


Solution:



Join QR.

i) In quadrilateral ORPQ,

$OQ \perp OP, OR \perp RP$
$\therefore \angle OQP = 90°, \angle ORP = 90°, \angle QPR = 60°$
$\angle QOR = 360° - (90° + 90° + 60°)$
$\angle QOR = 360° - 240°$
$\angle QOR = 120°$

ii) In ΔQOR,

OQ = QR (Radii of the same circle)

$\therefore \angle OQR = \angle QRO$..........(i)
but, $\angle OQR + \angle QRO + \angle QOR = 180°$
$\angle OQR + \angle QRO + 120° = 180°$
$\angle OQR + \angle QRO = 60°$
from (i)
$2\angle OQR = 60°$
$\angle OQR = 30°$

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iii) Now arc RQ subtends $\angle QOR$ at the centre and $\angle QSR$ at the remaining part of the circle.

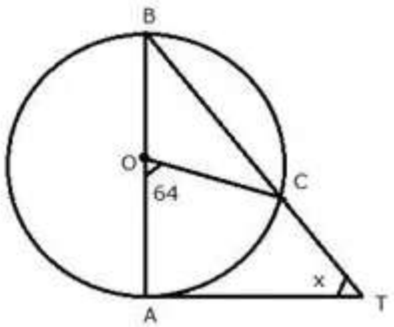
$\therefore \angle QSR = \frac{1}{2}\angle QOR$

$\Rightarrow \angle QSR = \frac{1}{2} \times 120°$

$\Rightarrow \angle QSR = 60°$

Question 19.

In the given figure, AB is a diameter of the circle, with centre O, and AT is a tangent. Calculate the numerical value of x.



Solution:

In ΔOBC,

OB = OC (Radii of the same circle)

$\therefore \angle OBC = \angle OCB$

But, Ext. $\angle COA = \angle OBC + \angle OCB$

Ext. $\angle COA = 2\angle OBC$

$\Rightarrow 64° = 2\angle OBC$

$\Rightarrow \angle OBC = 32°$

Now in ΔABT,

$\angle BAT = 90°$ $(OA \perp AT)$

$\angle OBC$ or $\angle ABT = 32°$

$\therefore \angle BAT + \angle ABT + x° = 180°$

$\Rightarrow 90° + 32° + x° = 180°$

$\Rightarrow x° = 58°$

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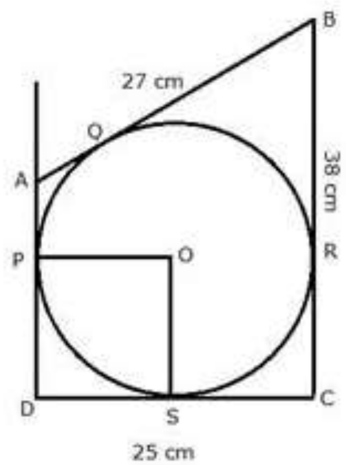
Question 20.
In quadrilateral ABCD, angle D = 90°, BC = 38 cm and DC = 25 cm. A circle is inscribed in this quadrilateral which touches AB at point Q such that QB = 27 cm. Find the radius of the circle.

Solution:



BQ and BR are the tangents from B to the circle.

Therefore, BR = BQ = 27 cm.

Also RC = (38 -; 27) = 11cm

Since CR and CS are the tangents from C to the circle

Therefore, CS = CR = 11 cm

So, DS = (25 - 11) = 14 cm

Now DS and DP are the tangents to the circle

Therefore, DS = DP

Now, $\angle PDS = 90^\circ$ (given)

and $OP \perp AD, OS \perp DC$

therefore, radius = DS = 14 cm

Question 21.
In the given figure, PT touches the circle with centre O at point R. Diameter SQ is produced to meet the tangent TR at P.
Given and ∠SPR = x° and ∠QRP = y°
Prove that -;

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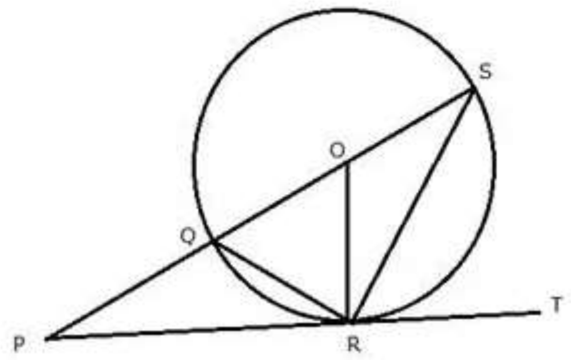
i) ∠ORS = y°
ii) write an expression connecting x and y



Solution:

$\angle QRP = \angle OSR = y$ (angles in alternate segment)

But OS = OR (Radii of the same circle)

$\therefore \angle ORS = \angle OSR = y$

$\therefore OQ = OR$ (radii of same circle)

$\therefore \angle OQR = \angle ORQ = 90^\circ - y$.........(i)(since $OR \perp PT$)

But in ΔPQR,

$Ext.\angle OQR = x + y$........(ii)

From (i) and (ii)

$x + y = 90^\circ - y$

$\Rightarrow x + 2y = 90^\circ$

Question 22.
PT is a tangent to the circle at T. If ; calculate:
i) ∠CBT
ii) ∠BAT
iii) ∠APT

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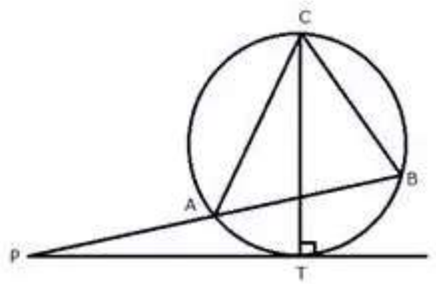
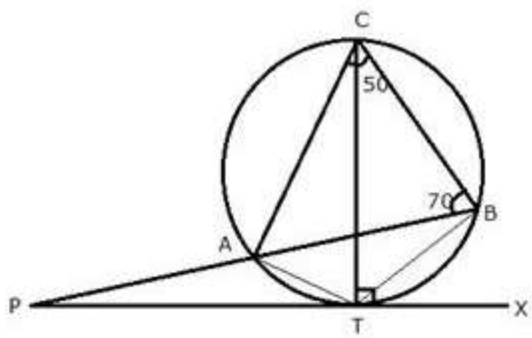


Solution:



Join AT and BT.

i) TC is the diameter of the circle

$\therefore \angle CBT = 90^\circ$ (Angle in a semi-circle)

ii) $\angle CBA = 70^\circ$

$\therefore \angle ABT = \angle CBT - \angle CBA = 90^\circ - 70^\circ = 20^\circ$

Now, $\angle ACT = \angle ABT = 20^\circ$ (Angles in the same segment of the circle)

$\therefore \angle TCB = \angle ACB - \angle ACT = 50^\circ - 20^\circ = 30^\circ$

But, $\angle TCB = \angle TAB$ (Angles in the same segment of the circle)

$\therefore \angle TAB$ or $\angle BAT = 30^\circ$

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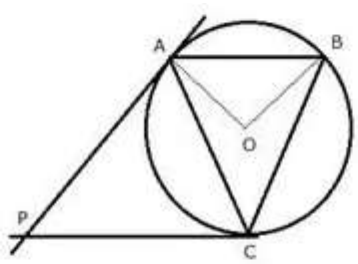
iii) $\angle BTX = \angle TCB = 30°$ (Angles in the same segment)

$\therefore \angle PTB = 180° - 30° = 150°$

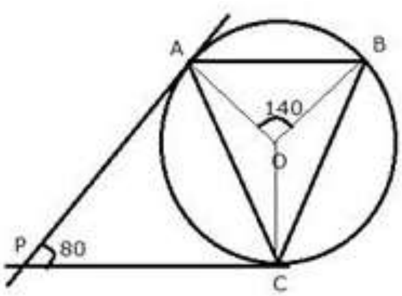
Now in ΔPTB,

$\angle APT + \angle PTB + \angle ABT = 180°$
$\Rightarrow \angle APT + 150° + 20° = 180°$
$\Rightarrow \angle APT = 180° - 170° = 10°$

Question 23.
In the given figure, O is the centre of the circumcircle ABC. Tangents at A and C intersect at P. Given angle AOB = 140° and angle APC = 80°; find the angle BAC.



Solution:



Join OC.

Therefore, PA and PA are the tangents

$\therefore OA \perp PA$ and $OC \perp PC$

In quadrilateral APCO,

$\angle APC + \angle AOC = 180°$
$\Rightarrow 80° + \angle AOC = 180°$
$\Rightarrow \angle AOC = 100°$

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$\angle BOC = 360° - (\angle AOB + \angle AOC)$
$\angle BOC = 360° - (140° + 100°)$
$\angle BOC = 360° - 240° = 120°$

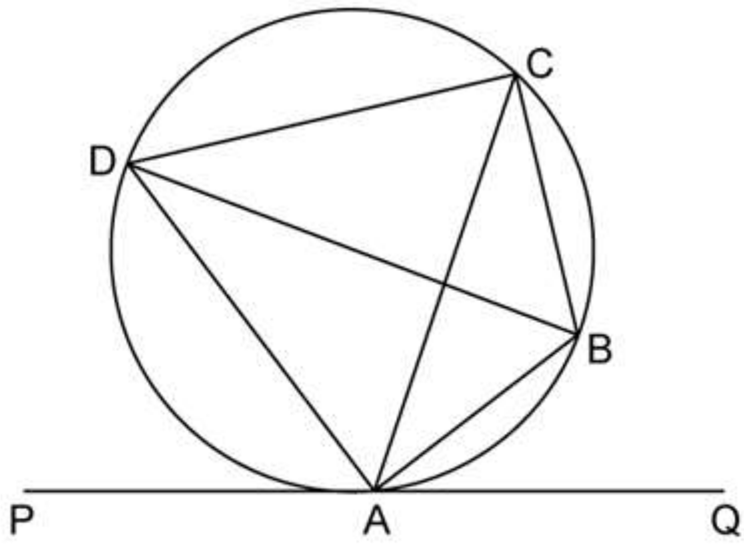
Now, arc BC subtends $\angle BOC$ at the centre and $\angle BAC$ at the remaining part of the circle

$\therefore \angle BAC = \frac{1}{2} \angle BOC$

$\angle BAC = \frac{1}{2} \times 120° = 60°$

Question 24.

In the given figure, PQ is a tangent to the circle at A. AB and AD are bisectors of ∠CAQ and ∠PAC. If ∠BAQ = 30°, prove that : BD is diameter of the circle.



Solution:

∠CAB = ∠BAQ = 30°……(AB is angle bisector of ∠CAQ)
∠CAQ = 2∠BAQ = 60°……(AB is angle bisector of ∠CAQ)
∠CAQ + ∠PAC = 180°……(angles in linear pair)
∴∠PAC = 120°
∠PAC = 2∠CAD……(AD is angle bisector of ∠PAC)
∠CAD = 60°

Now,
∠CAD + ∠CAB = 60 + 30 = 90°

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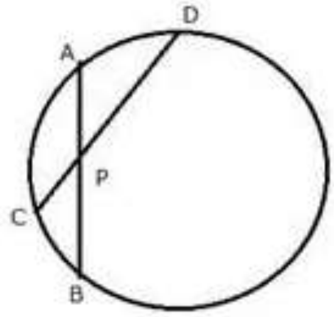
∠DAB = 90°
Thus, BD subtends 90° on the circle
So, BD is the diameter of circle

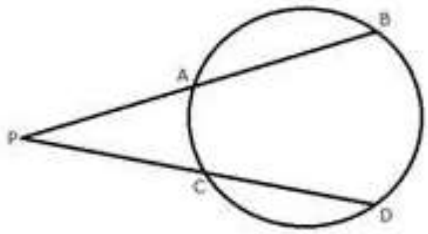
Exercise 18 B

Question 1.

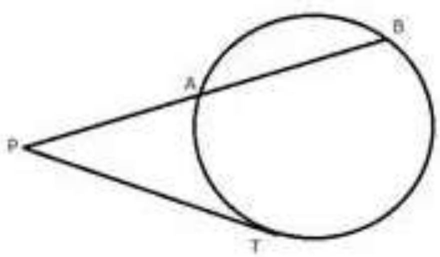
i) In the given figure, 3 x CP = PD = 9 cm and AP = 4.5 cm. Find BP.



ii) In the given figure, 5 x PA = 3 x AB = 30 cm and PC = 4cm. Find CD.



iii) In the given figure, tangent PT = 12.5 cm and PA = 10 cm; find AB.



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Solution:

i) Since two chords AB and CD intersect each other at P.

$\therefore AP \times PB = CP \times PD$

$\Rightarrow 4.5 \times PB = 3 \times 9 \quad (3CP = 9cm \Rightarrow CP = 3cm)$

$\Rightarrow PB = \dfrac{3 \times 9}{4.5} = 6 \text{ cm}$

ii) Since two chords AB and CD intersect each other at P.

$\therefore AP \times PB = CP \times PD$

But $5 \times PA = 3 \times AB = 30$ cm

$\therefore 5 \times PA = 30 \text{ cm} \Rightarrow PA = 6 \text{ cm}$

and $3 \times AB = 30 \text{ cm} \Rightarrow AB = 10 \text{ cm}$

$\Rightarrow BP = PA + AB = 6 + 10 = 16 \text{ cm}$

Now,

$AP \times PB = CP \times PD$

$\Rightarrow 6 \times 16 = 4 \times PD$

$\Rightarrow PD = \dfrac{6 \times 16}{4} = 24 \text{ cm}$

$CD = PD - PC = 24 - 4 = 20 \text{ cm}$

iii) Since PAB is the secant and PT is the tangent

$\therefore PT^2 = PA \times PB$

$\Rightarrow 12.5^2 = 10 \times PB$

$\Rightarrow PB = \dfrac{12.5 \times 12.5}{10} = 15.625 \text{ cm}$

$AB = PB - PA = 15.625 - 10 = 5.625 \text{ cm}$

Question 2.

In the given figure, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PB = 4 cm. Find

(i) AB.
(ii) the length of tangent PT.

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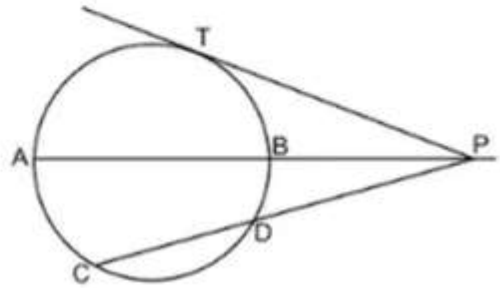


Solution:

(i) $PA = AB + BP = (AB + 4)$ cm

$PC = PD + CD = 5 + 7.8 = 12.8$ cm

Since $PA \times PB = PC \times PD$

$\Rightarrow (AB + 4) \times 4 = 12.8 \times 5$

$\Rightarrow AB + 4 = \dfrac{12.8 \times 5}{4}$

$\Rightarrow AB + 4 = 16$

$\Rightarrow AB = 12$ cm

(ii) Since $PT^2 = PC \times PD$

$\Rightarrow PT^2 = 12.8 \times 5$

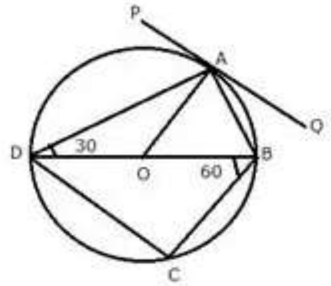
$\Rightarrow PT^2 = 64$

$\Rightarrow PT = 8$ cm

Question 3.

In the following figure, PQ is the tangent to the circle at A, DB is a diameter and O is the centre of the circle. If ; ∠ADB = 30° and ∠CBD = 60° calculate:

i) ∠QAD
ii) ∠PAD
iii) ∠CDB



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Solution:

i) PAQ is a tangent and AB is the chord.

$\angle QAB = \angle ADB = 30°$ (angles in the alternate segment)

ii) OA = OD (radii of the same circle)

$\therefore \angle OAD = \angle ODA = 30°$

But, $OA \perp PQ$

$\therefore \angle PAD = \angle OAP - \angle OAD = 90° - 30° = 60°$

iii) BD is the diameter.

$\therefore \angle BCD = 90°$ (angle in a semi-circle)

Now in ΔBCD,

$\angle CDB + \angle CBD + \angle BCD = 180°$

$\Rightarrow \angle CDB + 60° + 90° = 180°$

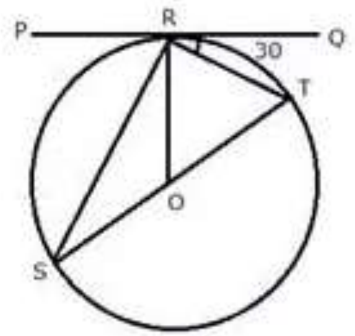
$\Rightarrow \angle CDB = 180° - 150° = 30°$

Question 4.

If PQ is a tangent to the circle at R; calculate:

i) ∠PRS

ii) ∠ROT



Given: O is the centre of the circle and ∠TRQ = 30°

Solution:

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PQ is a tangent and OR is the radius.

$\therefore$ OR $\perp$ PQ

$\therefore \angle ORT = 90^\circ$

$\Rightarrow \angle TRQ = 90^\circ - 30^\circ = 60^\circ$

But in ΔOTR,

OT = OR (Radii of the same circle)

$\therefore \angle OTR = 60^\circ$ or $\angle STR = 60^\circ$

But,

$\angle PRS = \angle STR = 60^\circ$ (angles in the alternate segment)

In ΔORT,

$\angle ORT = 60^\circ$

$\angle OTR = 60^\circ$

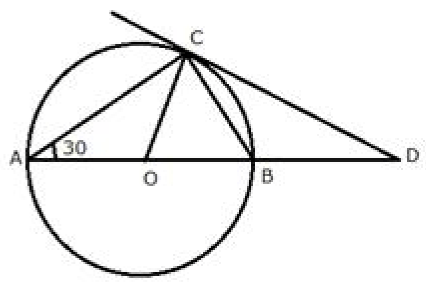
$\therefore \angle ROT = 180^\circ - (60^\circ + 60^\circ)$

$\angle ROT = 180^\circ - 120^\circ = 60^\circ$

Question 5.

AB is diameter and AC is a chord of a circle with centre O such that angle BAC=30º. The tangent to the circle at C intersects AB produced in D. Show that BC = BD.

Solution:



Join OC.

$\angle BCD = \angle BAC = 30^\circ$ (angles in alternate segment)

Arc BC subtends $\angle DOC$ at the centre of the circle and $\angle BAC$ at the remaining part of the circle.

$\therefore \angle BOC = 2\angle BAC = 2 \times 30^\circ = 60^\circ$

Now in ΔOCD,

$\angle BOC$ or $\angle DOC = 60^\circ$

$\angle OCD = 90^\circ$ (OC $\perp$ CD)

$\therefore \angle DOC + \angle ODC = 90^\circ$

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$\Rightarrow 60° + \angle ODC = 90°$

$\Rightarrow \angle ODC = 90° - 60° = 30°$

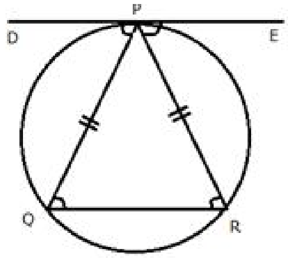
Now in ΔBCD,

$\because \angle ODC$ or $\angle BDC = \angle BCD = 30°$

$\therefore BC = BD$

Question 6.
Tangent at P to the circumcircle of triangle PQR is drawn. If this tangent is parallel to side QR, show that triangle PQR is isosceles.

Solution:



DE is the tangent to the circle at P.

DE||QR (Given)

$\angle EPR = \angle PRQ$ (Alternate angles are equal)

$\angle DPQ = \angle PQR$ (Alternate angles are equal)....(i)

Let $\angle DPQ = x$ and $\angle EPR = y$

Since the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment

$\therefore \angle DPQ = \angle PRQ$........(ii) (DE is tangent and PQ is chord)

from (i) and (ii)

$\angle PQR = \angle PRQ$

$\Rightarrow PQ = PR$

Hence, triangle PQR is an isosceles triangle.

Question 7.
Two circles with centers O and O' are drawn to intersect each other at points A and B. Centre O of one circle lies on the circumference of the other circle and CD is drawn tangent to the circle with centre O' at A. Prove that OA bisects angle BAC.

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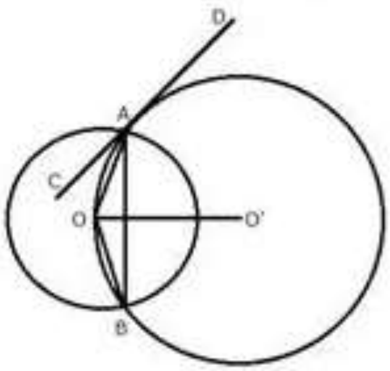
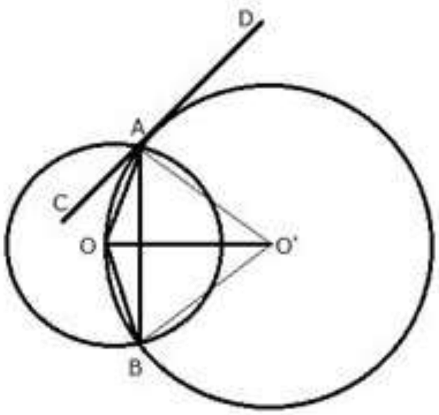


Solution:



Join OA, OB, O'A, O'B and O'O.

CD is the tangent and AO is the chord.

∠OAC = ∠OBA…….(i) (angles in alternate segment)

In ΔOAB,

OA = OB (Radii of the same circle)

∴ ∠OAB = ∠OBA………..(ii)

From (i) and (ii)

∠OAC = ∠OAB

Therefore, OA is bisector of ∠ BAC

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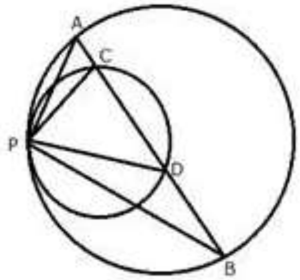
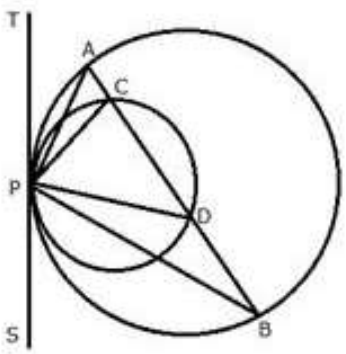
Question 8.
Two circles touch each other internally at a point P. A chord AB of the bigger circle intersects the other circle in C and D. Prove that: ∠CPA = ∠DPB



Solution:



Draw a tangent TS at P to the circles given.

Since TPS is the tangent, PD is the chord.

$\therefore \angle PAB = \angle BPS$..........(i)(angles in alternate segment)
Similarly,
$\angle PCD = \angle DPS$............(ii)

Subtracting (i) from (ii)

$\angle PCD - \angle PAB = \angle DPS - \angle BPS$

But in ΔPAC,

Ext.$\angle PCD = \angle PAB + \angle CPA$
$\therefore \angle PAB + \angle CPA - \angle PAB = \angle DPS - \angle BPS$
$\Rightarrow \angle CPA = \angle DPB$

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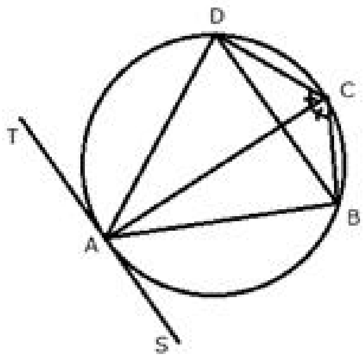
Question 9.
In a cyclic quadrilateral ABCD, the diagonal AC bisects the angle BCD. Prove that the diagonal BD is parallel to the tangent to the circle at point A.

Solution:



$\angle ADB = \angle ACB$..........(i)(angles in same segment)

Similarly,

$\angle ABD = \angle ACD$...........(ii)

But, $\angle ACB = \angle ACD$ (AC is bisector of $\angle BCD$)

$\therefore \angle ADB = \angle ABD$ (from (i) and (ii))

TAS is a tangent and AB is a chord

$\therefore \angle BAS = \angle ADB$ (angles in alternate segment)

But, $\angle ADB = \angle ABD$

$\therefore \angle BAS = \angle ABD$

But these are alternate angles

Therefore, TS||BD.

Question 10.
In the figure, ABCD is a cyclic quadrilateral with BC = CD. TC is tangent to the circle at point C and DC is produced to point G. If angle BCG = 108° and O is the centre of the circle, find:
i) angle BCT
ii) angle DOC

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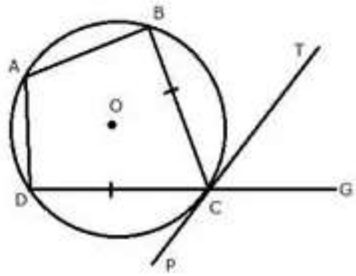
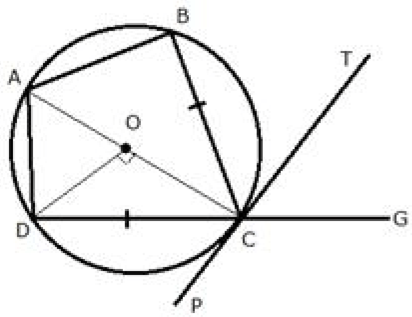


Solution:



Join OC, OD and AC.

i)

$\angle BCG + \angle BCD = 180^\circ$ (Linear pair)
$\Rightarrow 108^\circ + \angle BCD = 180^\circ$
$\Rightarrow \angle BCD = 180^\circ - 108^\circ = 72^\circ$

BC = CD
$\therefore \angle DCP = \angle BCT$
But, $\angle BCT + \angle BCD + \angle DCP = 180^\circ$
$\therefore \angle BCT + \angle BCT + 72^\circ = 180^\circ$
$2\angle BCT = 180^\circ - 72^\circ$
$\angle BCT = 54^\circ$

ii)

PCT is a tangent and CA is a chord.

$\therefore \angle CAD = \angle BCT = 54^\circ$

But arc DC subtends $\angle DOC$ at the centre and $\angle CAD$ at the remaining part of the circle.

$\therefore \angle DOC = 2\angle CAD = 2 \times 54^\circ = 108^\circ$

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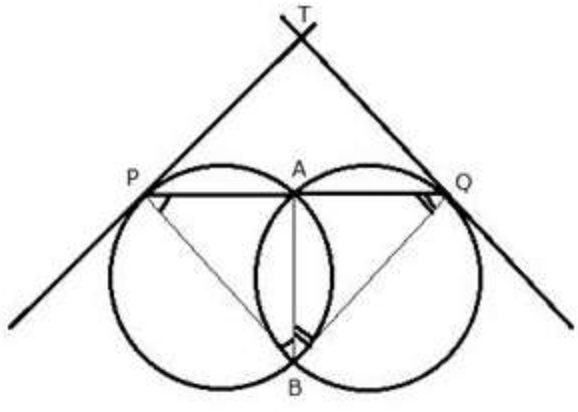
Question 11.
Two circles intersect each other at point A and B. A straight line PAQ cuts the circle at P and Q. If the tangents at P and Q intersect at point T; show that the points P, B, Q and T are concyclic.

Solution:



Join AB, PB and BQ

TP is the tangent and PA is a chord

$\therefore \angle TPA = \angle ABP$...........(i) (angles in alternate segment)

Similarly,

$\angle TQA = \angle ABQ$...........(ii)

Adding (i) and (ii)

$\angle TPA + \angle TQA = \angle ABP + \angle ABQ$
But, in ΔPTQ,
$\angle TPA + \angle TQA + \angle PTQ = 180°$
$\Rightarrow \angle PBQ = 180° - \angle PTQ$
$\Rightarrow \angle PBQ + \angle PTQ = 180°$

But they are the opposite angles of the quadrilateral

Therefore, PBQT are cyclic.

Hence, P, B, Q and T are concyclic.

Question 12.
In the figure, PA is a tangent to the circle. PBC is a secant and AD bisects angle BAC. Show that the triangle PAD is an isosceles triangle. Also show that:
$\angle CAD = \frac{1}{2}(\angle PBA - \angle PAB)$

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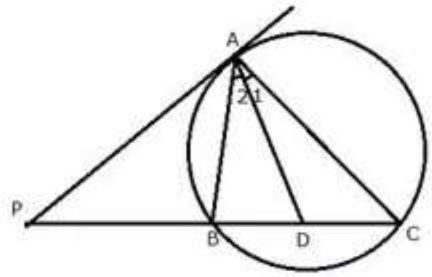

Solution:

i) PA is the tangent and AB is a chord

$\therefore \angle PAB = \angle C \ldots\ldots (i)$ (angles in the alternate segment)

AD is the bisector of $\angle BAC$

$\therefore \angle 1 = \angle 2 \ldots\ldots\ldots\ldots (ii)$

In ΔADC,

$$\text{Ext.}\angle ADP = \angle C + \angle 1$$
$$\Rightarrow \text{Ext.}\angle ADP = \angle PAB + \angle 2 = \angle PAD$$

Therefore, ΔPAD is an isosceles triangle.

ii) In ΔABC,

$$\text{Ext.}\angle PBA = \angle C + \angle BAC$$
$$\angle BAC = \angle PBA - \angle C$$
$$\Rightarrow \angle 1 + \angle 2 = \angle PBA - \angle PAB$$
(from (i) part)
$$2\angle 1 = \angle PBA - \angle PAB$$
$$\angle 1 = \frac{1}{2}(\angle PBA - \angle PAB)$$
$$\Rightarrow \angle CAD = \frac{1}{2}(\angle PBA - \angle PAB)$$

Question 13.
Two circles intersect each other at point A and B. Their common tangent touches the circles at points P and Q as shown in the figure. Show that the angles PAQ and PBQ are supplementary.

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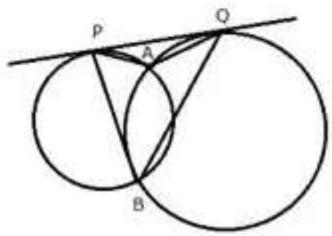
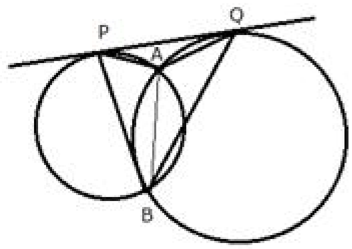


Solution:



Join AB.

PQ is the tangent and AB is a chord

$\therefore \angle QPA = \angle PBA$..........(i) (angles in alternate segment)

Similarly,

$\angle PQA = \angle QBA$..........(ii)

Adding (i) and (ii)

$\angle QPA + \angle PQA = \angle PBA + \angle QBA$
But, in ΔPAQ,
$\angle QPA + \angle PQA = 180° - \angle PAQ$............(iii)
and $\angle PBA + \angle QBA = \angle PBQ$..........(iv)

From (iii) and (iv)

$\angle PBQ = 180° - \angle PAQ$
$\Rightarrow \angle PBQ + \angle PAQ = 180°$
$\Rightarrow \angle PAQ + \angle PBQ = 180°$

Hence, $\angle PAQ$ and $\angle PBQ$ are supplementary.

Question 14.
In the figure, chords AE and BC intersect each other at point D.
i) if , $\angle CDE = 90°$ AB = 5 cm, BD = 4 cm and CD = 9 cm; find DE
ii) If AD = BD, Show that AE = BC.

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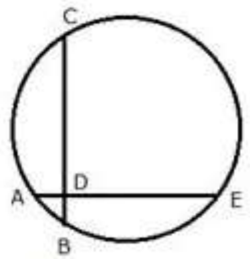

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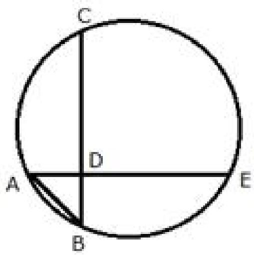

Join AB.

i) In Rt. ΔADB,

$AB^2 = AD^2 + DB^2$

$5^2 = AD^2 + 4^2$

$AD^2 = 25 - 16$

$AD^2 = 9$

$AD = 3$

Chords AE and CB intersect each other at D inside the circle

AD x DE = BD x DC

3 x DE = 4 x 9

DE = 12 cm

ii) If AD = BD ....(i)

We know that:

AD x DE = BD x DC

But AD = BD

Therefore, DE = DC ....(ii)

Adding (i) and (ii)

AD + DE = BD + DC

Therefore, AE = BC

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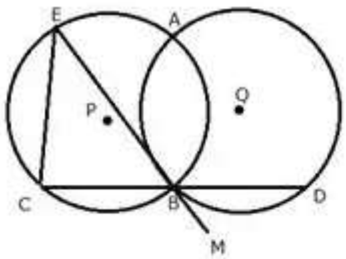
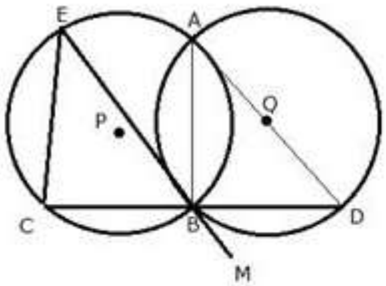
Question 15.
Circles with centers P and Q intersect at points A and B as shown in the figure. CBD is a line segment and EBM is tangent to the circle, with centre Q, at point B. If the circles are congruent; show that CE = BD.



Solution:



Join AB and AD

EBM is a tangent and BD is a chord.

$\angle DBM = \angle BAD$ (angles in alternate segments)

But, $\angle DBM = \angle CBE$ (Vertically opposite angles)

$\therefore \angle BAD = \angle CBE$

Since in the same circle or congruent circles, if angles are equal, then chords opposite to them are also equal.

Therefore, CE = BD

Question 16.
In the adjoining figure, O is the centre of the circle and AB is a tangent to it at point B.
Find ∠BDC = 65. Find ∠BAO

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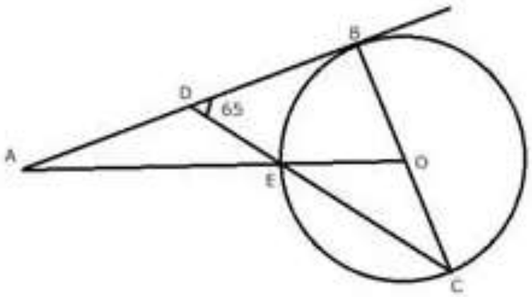

Solution:

AB is a straight line.

$\therefore \angle ADE + \angle BDE = 180^\circ$
$\Rightarrow \angle ADE + 65^\circ = 180^\circ$
$\Rightarrow \angle ADE = 115^\circ$..........(i)

AB i.e. DB is tangent to the circle at point B and BC is the diameter.

$\therefore \angle DBC = 90^\circ$
In ΔBDC,
$\angle DBC + \angle BDC + \angle DCB = 180^\circ$
$\Rightarrow 90^\circ + 65^\circ + \angle DCB = 180^\circ$
$\Rightarrow \angle DCB = 25^\circ$

Now, OE = OC (radii of the same circle)

$\therefore \angle DCB$ or $\angle OCE = \angle OEC = 25^\circ$
Also,
$\angle OEC = \angle DEA = 25^\circ$..........(ii)

(vertically opposite angles)

In ΔADE,

$\angle ADE + \angle DEA + \angle DAE = 180^\circ$
From (i) and (ii)
$115^\circ + 25^\circ + \angle DAE = 180^\circ$
$\Rightarrow \angle DAE$ or $\angle BAO = 180^\circ - 140^\circ = 40^\circ$
$\therefore \angle BAO = 40^\circ$

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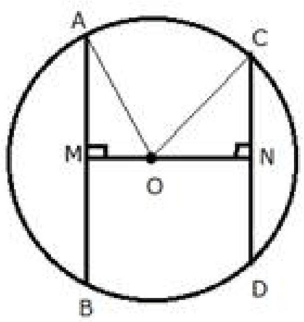
Exercise 18 C

Question 1.
Prove that of any two chord of a circle, the greater chord is nearer to the centre.

Solution:



Given: A circle with centre O and radius r. $OM \perp AB$ and $ON \perp CD$. Also AB > CD

To prove: OM < ON

Proof: Join OA and OC.

In Rt.ΔAOM,

$$AO^2 = AM^2 + OM^2$$

$$\Rightarrow r^2 = \left(\frac{1}{2}AB\right)^2 + OM^2$$

$$\Rightarrow r^2 = \frac{1}{4}AB^2 + OM^2 \text{............(i)}$$

Again in Rt. ΔONC,

$$OC^2 = NC^2 + ON^2$$

$$\Rightarrow r^2 = \left(\frac{1}{2}CD\right)^2 + ON^2$$

$$\Rightarrow r^2 = \frac{1}{4}CD^2 + ON^2 \text{............(ii)}$$

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$$\Rightarrow r^2 = \frac{1}{4}CD^2 + ON^2 \ldots\ldots\ldots\ldots(ii)$$

From (i) and (ii)

$$\frac{1}{4}AB^2 + OM^2 = \frac{1}{4}CD^2 + ON^2$$

But, $AB > CD$ (given)

$\therefore ON > OM$

$\Rightarrow OM < ON$

Hence, AB is nearer to the centre than CD.

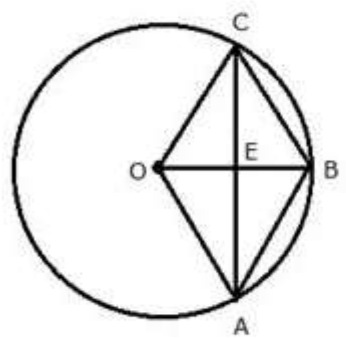
Question 2.

OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O.

i) If the radius of the circle is 10 cm, find the area of the rhombus.

ii) If the area of the rhombus is $32\sqrt{3}$ cm^2, find the radius of the circle.

Solution:



i) Radius = 10 cm

In rhombus OABC,

OC = 10 cm

$$\therefore OE = \frac{1}{2} \times OB = \frac{1}{2} \times 10 = 5 \text{ cm}$$

In Rt.ΔOCE,

$$OC^2 = OE^2 + EC^2$$

$$\Rightarrow 10^2 = 5^2 + EC^2$$

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$\Rightarrow EC = 5\sqrt{3}$

$\therefore AC = 2 \times EC = 2 \times 5\sqrt{3} = 10\sqrt{3}$

Area of rhombus = $\frac{1}{2} \times OB \times AC$

$= \frac{1}{2} \times 10 \times 10\sqrt{3}$

$= 50\sqrt{3}\ cm^2 \approx 86.6\ cm^2\ \ (\sqrt{3} = 1.73)$

ii) Area of rhombus = $32\sqrt{3}\ cm^2$

But area of rhombus OABC = 2 x area of ΔOAB

Area of rhombus OABC = $2 \times \frac{\sqrt{3}}{4} r^2$

Where r is the side of the equilateral triangle OAB.

$2 \times \frac{\sqrt{3}}{4} r^2 = 32\sqrt{3}$

$\Rightarrow \frac{\sqrt{3}}{2} r^2 = 32\sqrt{3}$

$\Rightarrow r^2 = 64$

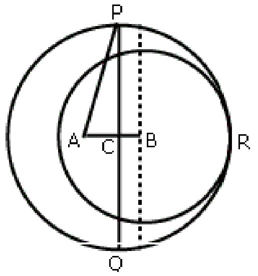
$\Rightarrow r = 8$

Therefore, radius of the circle = 8 cm

Question 3.

Two circles with centers A and B, and radii 5 cm and 3 cm, touch each other internally. If the perpendicular bisector of the segment AB meets the bigger circle in P and Q; find the length of PQ.

Solution:



If two circles touch internally, then distance between their centres is equal to the difference of their radii. So, AB = (5 - 3) cm = 2 cm.

Also, the common chord PQ is the perpendicular bisector of AB. Therefore, AC = CB = $\frac{1}{2}$ AB = 1 cm

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In right Δ ACP, we have $AP^2 = AC^2 + CP^2$

$\Rightarrow 5^2 = 1^2 + CP^2$

$\Rightarrow CP^2 = 25 -; 1 = 24$

$\Rightarrow CP = \sqrt{24} = 2\sqrt{6}$ cm

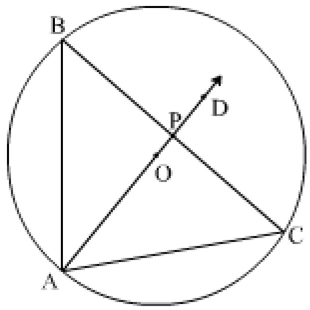
Now, PQ = 2 CP

$= 2 \times 2\sqrt{6}$ cm

$= 4\sqrt{6}$ cm

Question 4.
Two chords AB and AC of a circle are equal. Prove that the centre of the circle, lies on the bisector of the angle BAC.

Solution:



Given: AB and AC are two equal chords of C (O, *r*).

To prove: Centre, O lies on the bisector of $\angle$ BAC.

Construction: Join BC. Let the bisector of $\angle$ BAC intersects BC in P.

Proof:

In Δ APB and Δ APC,

AB = AC (Given)

$\angle$ BAP = $\angle$ CAP (Given)

AP = AP (Common)

$\therefore \Delta APB \cong \Delta APC$ (SAS congruence criterion)

$\Rightarrow$ BP = CP and $\angle$ APB = $\angle$ APC (CPCT)

$\angle APB + \angle APC = 180°$ (Linear pair)

=> $2\angle APB = 180°$ ($\angle APB = \angle APC$)

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$\Rightarrow \angle APB = 90°$

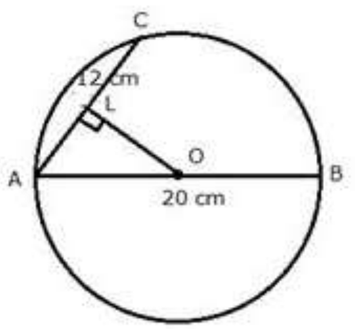
Now, BP = CP and $\angle APB = 90°$

$\therefore$ AP is the perpendicular bisector of chord BC.

$\Rightarrow$ AP passes through the centre, O of the circle.

Question 5.
The diameter and a chord of circle have a common end-point. If the length of the diameter is 20 cm and the length of the chord is 12 cm, how far is the chord from the centre of the circle?

Solution:



AB is the diameter and AC is the chord.

Draw $OL \perp AC$

Since $OL \perp AC$ and hence it bisects AC, O is the centre of the circle.

Therefore, OA = 10 cm and AL = 6 cm

Now, in Rt. ΔOLA,

$$AO^2 = AL^2 + OL^2$$
$$\Rightarrow 10^2 = 6^2 + OL^2$$
$$\Rightarrow OL^2 = 100 - 36 = 64$$
$$\Rightarrow OL = 8 \text{ cm}$$

Therefore, chord is at a distance of 8 cm from the centre of the circle.

Question 6.
ABCD is a cyclic quadrilateral in which BC is parallel to AD, angle ADC = 110° and angle BAC = 50°. Find angle DAC and angle DCA.

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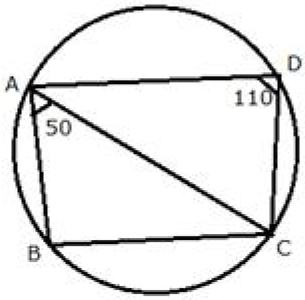
Solution:



ABCD is a cyclic quadrilateral in which AD||BC

$\angle ADC = 110^\circ, \angle BAC = 50^\circ$
$\angle B + \angle D = 180^\circ$

(Sum of opposite angles of a quadrilateral)

$\Rightarrow \angle B + 110^\circ = 180^\circ$
$\Rightarrow \angle B = 70^\circ$

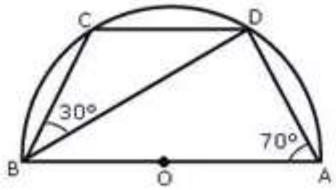
Now in ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$
$\Rightarrow 50^\circ + 70^\circ + \angle ACB = 180^\circ$
$\Rightarrow \angle ACB = 180^\circ - 120^\circ = 60^\circ$
$\because AD \parallel BC$
$\therefore \angle DAC = \angle ACB = 60^\circ$ (alternate angles)

Now in ΔADC,

$\angle DAC + \angle ADC + \angle DCA = 180^\circ$
$\Rightarrow 60^\circ + 110^\circ + \angle DCA = 180^\circ$
$\Rightarrow \angle DCA = 180^\circ - 170^\circ = 10^\circ$

Question 7.
In the given figure, C and D are points on the semicircle described on AB as diameter. Given angle BAD = 70° and angle DBC = 30°, calculate angle BDC



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Solution:

Since ABCD is a cyclic quadrilateral, therefore, ∠BCD + ∠BAD = 180°

(since opposite angles of a cyclic quadrilateral are supplementary)

⇒ ∠BCD + 70° = 180°

⇒ ∠BCD = 180° - 70° = 110°

In △BCD, we have,

∠CBD + ∠BCD + ∠BDC = 180°

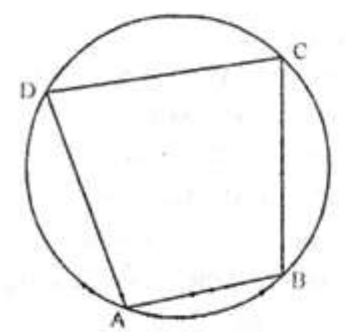
⇒30° + 110° + ∠BDC = 180°

⇒ ∠BDC = 180° - 140°

⇒ ∠BDC = 40°

Question 8.
In cyclic quadrilateral ABCD, ∠A = 3 ∠C and ∠D = 5∠B. Find the measure of each angle of the quadrilateral.

Solution:



ABCD is a cyclic quadrilateral.
∴ ∠A + ∠C = 180°
⇒ 3∠C + ∠C = 180°
⇒ 4∠C = 180°
⇒ ∠C = 45°

∵ ∠A = 3∠C
⇒ ∠A = 3 × 45°
⇒ ∠A = 135°
Similarly,

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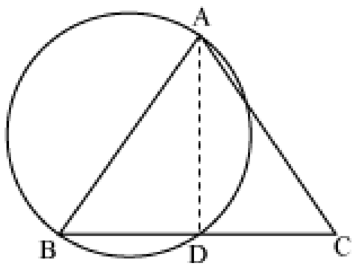
$\therefore \angle B + \angle D = 180°$
$\Rightarrow \angle B + 5\angle B = 180°$
$\Rightarrow 6\angle B = 180°$
$\Rightarrow \angle B = 30°$

$\because \angle D = 5\angle B$
$\Rightarrow \angle D = 5 \times 30°$ >
$\Rightarrow \angle D = 150°$
Hence, $\angle A = 1350$, $\angle B = 30°$, $\angle C = 450$, $\angle D = 150°$

Question 9.
Show that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.

Solution:



Join AD.

AB is the diameter.

$\therefore \angle ADB = 90°$ (Angle in a semi-circle)

But, $\angle ADB + \angle ADC = 180°$ (linear pair)

$\Rightarrow \angle ADC = 90°$

In ΔABD and ΔACD,

$\angle ADB = \angle ADC$ (each 90°)

AB = AC (Given)

AD = AD (Common)

$\therefore \Delta ABD \cong \Delta ACD$ (RHS congruence criterion)

$\Rightarrow$ BD = DC (C.P.C.T)

Hence, the circle bisects base BC at D.

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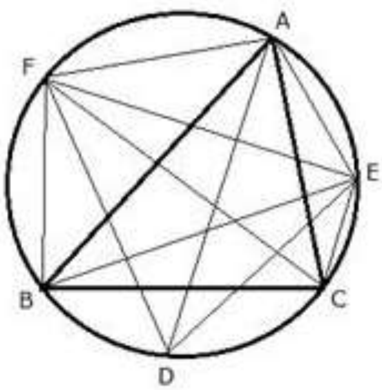
Question 10.
Bisectors of vertex A, B and C of a triangle ABC intersect its circumcircle at points D, E and F respectively. Prove that angle EDF = $90^\circ - \frac{1}{2}\angle A$

Solution:



Join ED, EF and DF. Also join BF, FA, AE and EC.

$\angle EBF = \angle ECF = \angle EDF$..........(i) (angles in the same segment)

In cyclic quadrilateral AFBE,

$\angle EBF + \angle EAF = 180^\circ$...........(ii) (Sum of opposite angles)

Similarly in cyclic quadrilateral CEAF,

$\angle EAF + \angle ECF = 180^\circ$...........(iii)

Adding (ii) and (iii)

$\angle EBF + \angle ECF + 2\angle EAF = 360^\circ$
$\Rightarrow \angle EDF + \angle EDF + 2\angle EAF = 360^\circ$ (from (i))
$\Rightarrow 2\angle EDF + 2\angle EAF = 360^\circ$
$\Rightarrow \angle EDF + \angle EAF = 180^\circ$
$\Rightarrow \angle EDF + \angle 1 + \angle BAC + \angle 2 = 180^\circ$
But $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$

(angles in the same segment)

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$\therefore \angle EDF + \angle 3 + \angle BAC + \angle 4 = 180°$

But $\angle 4 = \frac{1}{2}\angle C, \angle 3 = \frac{1}{2}\angle B$

$\therefore \angle EDF + \frac{1}{2}\angle B + \angle BAC + \frac{1}{2}\angle C = 180°$

$\Rightarrow \angle EDF + \frac{1}{2}\angle B + 2 \times \frac{1}{2}\angle A + \frac{1}{2}\angle C = 180°$

$\Rightarrow \angle EDF + \frac{1}{2}(\angle A + \angle B + \angle C) + \frac{1}{2}\angle A = 180°$

$\Rightarrow \angle EDF + \frac{1}{2}(180°) + \frac{1}{2}\angle A = 180°$

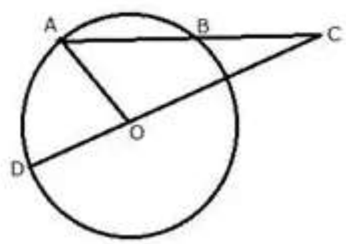
$\Rightarrow \angle EDF + 90° + \frac{1}{2}\angle A = 180°$

$\Rightarrow \angle EDF = 180° - \left(90° + \frac{1}{2}\angle A\right)$

$\Rightarrow \angle EDF = 180° - 90° \frac{1}{2}\angle A$

$\Rightarrow \angle EDF = 90° - \frac{1}{2}\angle A$

Question 11.
In the figure, AB is the chord of a circle with centre O and DOC is a line segment such that BC = DO. If ∠C = 20°, find angle AOD.



Solution:

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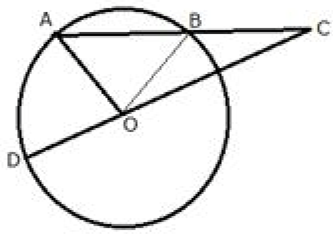

Join OB.

In ΔOBC,

$BC = OD = OB$ (Radii of the same circle)
$\therefore \angle BOC = \angle BCO = 20°$
and Ext.$\angle ABO = \angle BCO + \angle BOC$
$\Rightarrow$ Ext.$\angle ABO = 20° + 20° = 40°$............(i)

In ΔOAB,

$OA = OB$ (Radii of the same circle)
$\therefore \angle OAB = \angle OBA = 40°$ (from (i))
$\angle AOB = 180° - \angle OAB - \angle OBA$
$\Rightarrow \angle AOB = 180° - 40° - 40° = 100°$

Since DOC is a straight line

$\therefore \angle AOD + \angle AOB + \angle BOC = 180°$
$\Rightarrow \angle AOD + 100° + 20° = 180°$
$\Rightarrow \angle AOD = 180° - 120°$
$\Rightarrow \angle AOD = 60°$

Question 12.
Prove that the perimeter of a right triangle is equal to the sum of the diameter of its in circle and twice the diameter of its circumcircle.

Solution:

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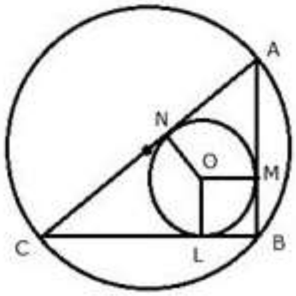

Join OL, OM and ON.

Let D and d be the diameter of the circumcircle and incircle.

and let R and r be the radius of the circumcircle and incircle.

In circumcircle of ΔABC,

$\angle B = 90°$

Therefore, AC is the diameter of the circumcircle i.e. AC = D

Let radius of the incircle = r

$\therefore$ OL = OM = ON = r

Now, from B, BL, BM are the tangents to the incircle.

$\therefore$ BL = BM = r

Similarly,

AM = AN and CL = CN = R

(Tangents from the point outside the circle)

Now,

AB+BC+CA = AM+BM+BL+CL+CA

= AN+r+r+CN+CA

= AN+CN+2r+CA

= AC+AC+2r

= 2AC+2r

= 2D+d

Question 13.
P is the midpoint of an arc APB of a circle. Prove that the tangent drawn at P will be parallel to the chord AB.

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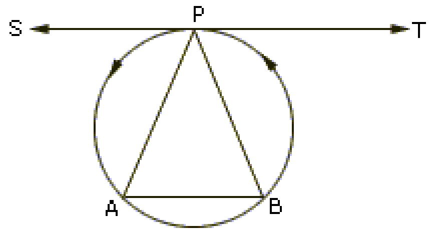
Solution:



Join AP and BP.

Since TPS is a tangent and PA is the chord of the circle.

$\angle BPT = \angle PAB$ (angles in alternate segments)

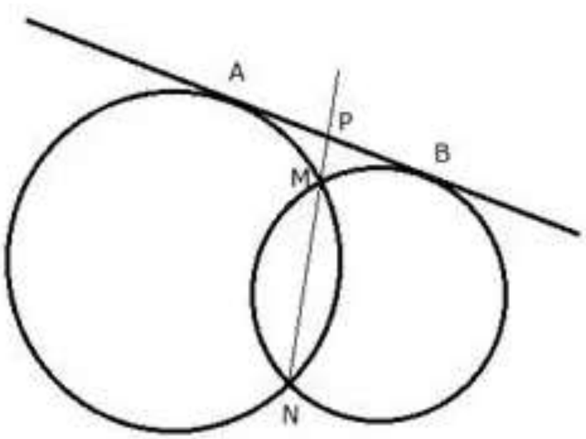
But

$\angle PBA = \angle PAB (\because PA = PB)$
$\therefore \angle BPT = \angle PBA$

But these are alternate angles

$\therefore$ TPS || AB

Question 14.
In the given figure, MN is the common chord of two intersecting circles and AB is their common tangent.



Prove that the line NM produced bisects AB at P.

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Solution:

From P, AP is the tangent and PMN is the secant for first circle.

$\therefore\ AP^2 = PM \times PN$.......(i)

Again from P, PB is the tangent and PMN is the secant for second circle.

$\therefore\ PB^2 = PM \times PN$. ....(ii)

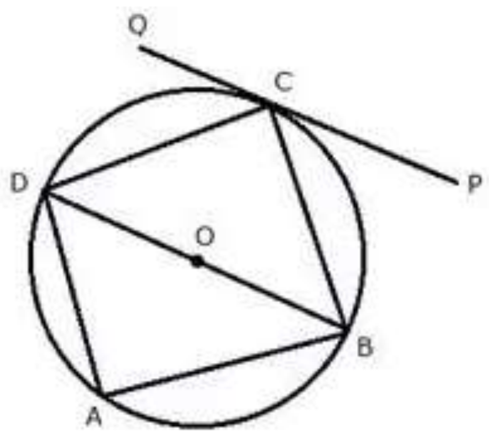
From (i) and (ii)

$AP^2 = PB^2$
$\Rightarrow AP = PB$

Therefore, P is the midpoint of AB.

Question 15.
In the given figure, ABCD is a cyclic quadrilateral, PQ is tangent to the circle at point C and BD is its diameter. If ∠DCQ = 40° and ∠ABD = 60°, find:

i) ∠DBC
ii) ∠ BCP
iii) ∠ ADB



Solution:

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i) PQ is tangent and CD is a chord

$\therefore \angle DCQ = \angle DBC$ (angles in the alternate segment)

$\therefore \angle DBC = 40^\circ \quad (\because \angle DCQ = 40^\circ)$

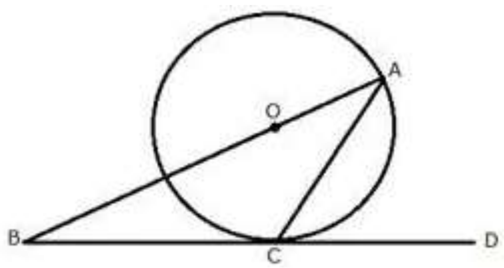
ii)

$\angle DCQ + \angle DCB + \angle BCP = 180^\circ$
$\Rightarrow 40^\circ + 90^\circ + \angle BCP = 180^\circ \quad (\because \angle DCB = 90^\circ)$
$\Rightarrow \angle BCP = 180^\circ - 130^\circ = 50^\circ$

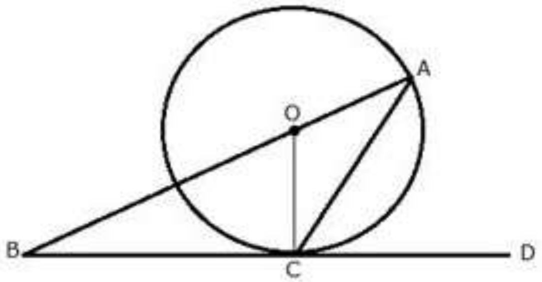
iii) In ΔABD,

$\angle BAD = 90^\circ, \angle ABD = 60^\circ$
$\therefore \angle ADB = 180^\circ - (90^\circ + 60)$
$\Rightarrow \angle ADB = 180^\circ - 150^\circ = 30^\circ$

Question 16.
The given figure shows a circle with centre O and BCD is a tangent to it at C. Show that: ∠ACD + ∠BAC = 90°



Solution:



Join OC.

BCD is the tangent and OC is the radius.

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$\therefore\ OC \perp BD$

$\Rightarrow \angle OCD = 90^\circ$

$\Rightarrow \angle OCA + \angle ACD = 90^\circ$............(i)

But in ΔOCA

$OA = OC$ (radii of same circle)

$\therefore\ \angle OCA = \angle OAC$

Substituting in (i)

$\angle OAC + \angle ACD = 90^\circ$

$\Rightarrow \angle BAC + \angle ACD = 90^\circ$

Question 17.

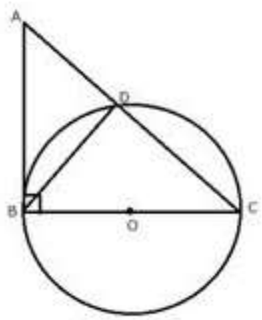
ABC is a right triangle with angle B = 90º. A circle with BC as diameter meets by hypotenuse AC at point D.

Prove that –

i) $AC \times AD = AB^2$

ii) $BD^2 = AD \times DC$.

Solution:



i) In ΔABC,

$\angle B = 90^\circ$ and BC is the diameter of the circle.

Therefore, AB is the tangent to the circle at B.

Now, AB is tangent and ADC is the secant

$\therefore\ AB^2 = AD \times AC$

ii) In ΔADB,

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$\angle D = 90^\circ$

$\therefore \angle A + \angle ABD = 90^\circ$.......(i)

But in ΔABC, $\angle B = 90^\circ$

$\therefore \angle A + \angle C = 90^\circ$.........(ii)

From (i) and (ii)

$\angle C = \angle ABD$

Now in ΔABD and ΔCBD,

$\angle BDA = \angle BDC = 90^\circ$

$\angle ABD = \angle BCD$

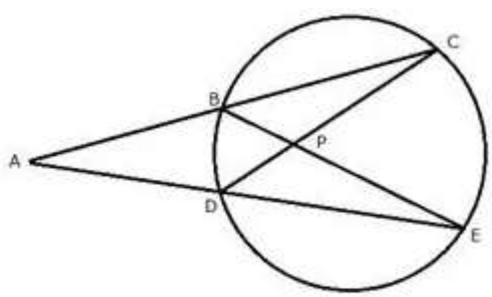
$\therefore \Delta ABD \sim \Delta CBD$ (AA Postulate)

$$\therefore \frac{BD}{DC} = \frac{AD}{BD}$$

$\Rightarrow BD^2 = AD \times DC$

Question 18.
In the given figure AC = AE.

Show that:
i) CP = EP
ii) BP = DP



Solution:

In ΔADC and ΔABE,

$\angle ACD = \angle AEB$ (angles in the same segment)

AC = AE (Given)

$\angle A = \angle A$ (Common)

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$\therefore \Delta ADC \cong \Delta ABE$ (ASA Postulate)

$\Rightarrow AB = AD$

but AC = AE

$\therefore AC - AB = AE - AD$

$\Rightarrow BC = DE$

In ΔBPC and ΔDPE,

$\angle C = \angle E$ (angles in the same segment)

BC = DE

$\angle CBP = \angle CDE$ (angles in the same segment)

$\therefore \Delta BPC \cong \Delta DPE$ (ASA Postulate)

$\Rightarrow BP = DP$ and $CP = PE$ (cpct)

Question 19.

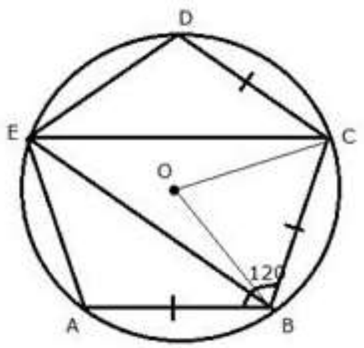
ABCDE is a cyclic pentagon with centre of its circumcircle at point O such that AB = BC = CD and angle ABC=120°

Calculate:

i) ∠BEC

ii) ∠ BED

Solution:



i) Join OC and OB.

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AB = BC = CD and $\angle ABC = 120°$

$\therefore \angle BCD = \angle ABC = 120°$

OB and OC are the bisectors of $\angle ABC$ and $\angle BCD$ respectively.

$\therefore \angle OBC = \angle BCO = 60°$

In ΔBOC,

$\angle BOC = 180° - (\angle OBC + \angle BOC)$
$\Rightarrow \angle BOC = 180° - (60° + 60°)$
$\Rightarrow \angle BOC = 180° - 120° = 60°$

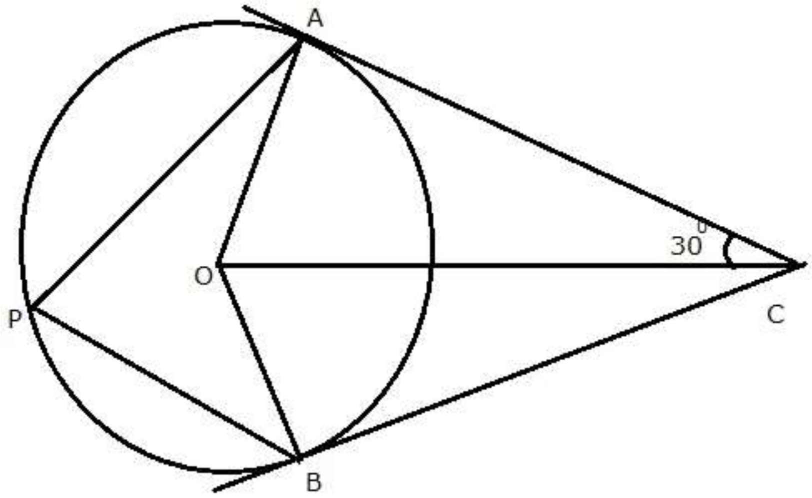
Arc BC subtends $\angle BOC$ at the centre and $\angle BEC$ at the remaining part of the circle.

$\therefore \angle BEC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 60° = 30°$

ii) In cyclic quadrilateral BCDE,

$\angle BED + \angle BCD = 180°$
$\Rightarrow \angle BED + 120° = 180°$
$\therefore \angle BED = 60°$

Question 20.
In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If angle ACO = 30°, find:
(i) angle BCO
(ii) angle AOB
(iii) angle APB



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Solution:

In the given fig, O is the centre of the circle and CA and CB are the tangents to the circle from C. Also, $\angle ACO = 30°$

P is any point on the circle. P and PB are joined.

To find: (i) $\angle BCO$

(ii) $\angle AOB$

(iii) $\angle APB$

Proof:

(i) In ΔOAC and OBC,

$OC = OC$ (common)

$OA = OB$ (radius of the circle)

$CA = CB$ (tangents to the circle)

$\therefore \Delta OAC \cong \Delta OBC$ (SSS congruence criterion)

$\therefore \angle ACO = \angle BCO = 30°$

(ii) $\therefore \angle ACB = 30° + 30° = 60°$

$\therefore \angle AOB + \angle ACB = 180°$

$\Rightarrow \angle AOB + 60° = 180°$

$\Rightarrow \angle AOB = 180° - 60°$

$\Rightarrow \angle AOB = 120°$

(iii) Arc AB subtends $\angle AOB$ at the centre and $\angle APB$ is in the remaining part of the circle

$\therefore \angle APB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 120° = 60°$

Question 21.

ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.

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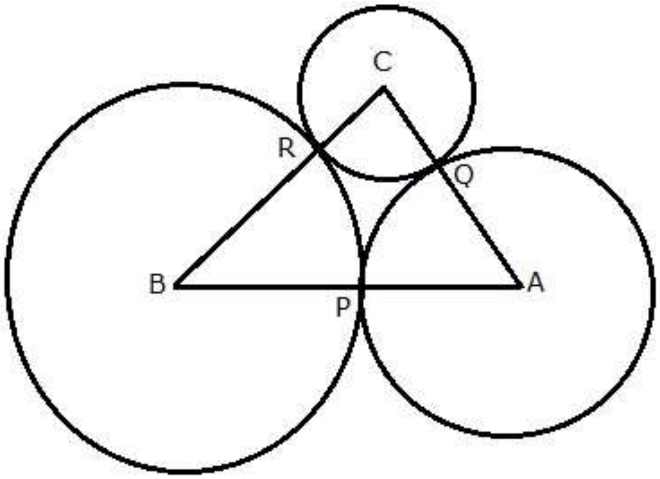
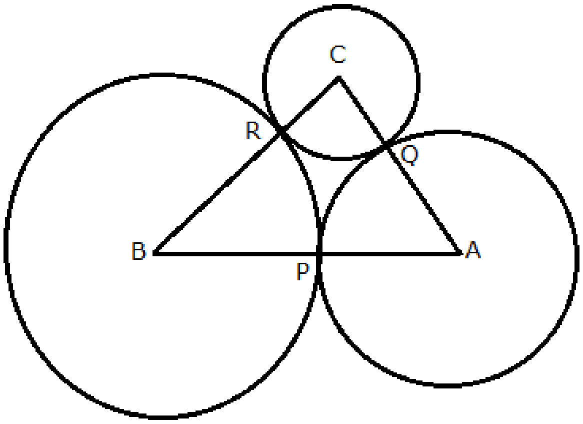


Solution:



Given: ABC is a triangle with AB = 10 cm, BC= 8 cm, AC = 6 cm. Three circles are drawn with centre A, B and C touch each other at P, Q and R respectively.

We need to find the radii of the three circles.

Let
PA = AQ = x
QC = CR = y
RB = BP = z
$\therefore$ x+z=10 .....(1)
z+y=8 .....(2)
y+x=6 .....(3)

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Adding all the three equations, we have

$2(x+y+z) = 24$

$\Rightarrow x+y+z=\frac{24}{2} = 12$ .....(4)

Subtracting (1), (2) and (3) from (4)

$y=12-10=2$

$x = 12-8=4$

$z = 12-6=6$

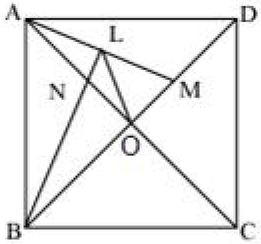
Therefore, radii are 2 cm, 4 cm and 6 cm

Question 22.

In a square ABCD, its diagonal AC and BD intersect each other at point O. The bisector of angle DAO meets BD at point M and bisector of angle ABD meets AC at N and AM at L. Show that –

i) ∠ONL + ∠OML = 180°
ii) ∠BAM = ∠BMA
iii) ALOB is a cyclic quadrilateral.

Solution:



i) $\therefore \angle AOB = \angle AOD = 90°$

In ΔANB,

$\angle ANB = 180° - (\angle NAB + \angle NBA)$

$\Rightarrow \angle ANB = 180° - \left(45° + \frac{45°}{2}\right)$ (NB is bisector of ∠ABD)

$\Rightarrow \angle ANB = 180° - 45° - \frac{45°}{2} = 135° - \frac{45°}{2}$

But, $\angle LNO = \angle ANB$ (vertically opposite angles)

$\therefore \angle LNO = 135° - \frac{45°}{2}$..........(i)

Now in ΔAMO,

$\angle AMO = 180° - (\angle AOM + \angle OAM)$

$\Rightarrow \angle AMO = 180° - (90° + \frac{45°}{2})$ (MA is bisector of ∠DAO)

$\Rightarrow \angle AMO = 180° - 90° - \frac{45°}{2} = 90° - \frac{45°}{2}$.......(ii)

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Adding (i) and (ii)

$$\angle LNO + \angle AMO = 135° - \frac{45°}{2} + 90° - \frac{45°}{2}$$

$$\Rightarrow \angle LNO + \angle AMO = 225° - 45° = 180°$$

$$\Rightarrow \angle ONL + \angle OML = 180°$$

ii)

$$\angle BAM = \angle BAO + \angle OAM$$

$$\Rightarrow \angle BAM = 45° + \frac{45°}{2} = 67\frac{1°}{2}$$

and

$$\angle BMA = 180° - (\angle AOM + \angle OAM)$$

$$\Rightarrow \angle BMA = 180° - 90° - \frac{45°}{2} = 90° - \frac{45°}{2} = 67\frac{1°}{2}$$

$$\therefore \angle BAM = \angle BMA$$

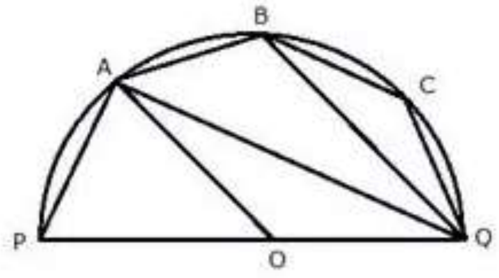
iii) In quadrilateral ALOB,

$$\because \angle ABO + \angle ALO = 45° + 90° + 45° = 180°$$

Therefore, ALOB is a cyclic quadrilateral.

Question 23.

The given figure shows a semicircle with centre O and diameter PQ. If PA = AB and ∠BOQ = 140°; find measures of angles PAB and AQB. Also, show that AO is parallel to BQ.



Solution:

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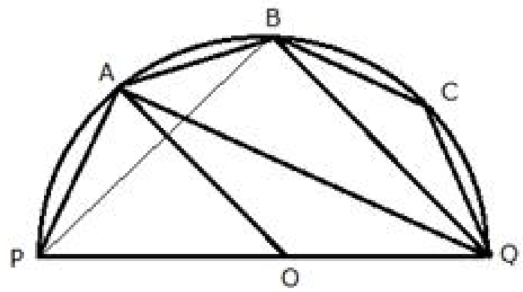

Join PB.

i) In cyclic quadrilateral PBCQ,

$$\angle BPQ + \angle BCQ = 180^\circ$$
$$\Rightarrow \angle BPQ + 140^\circ = 180^\circ$$
$$\Rightarrow \angle BPQ = 40^\circ \quad \ldots.(1)$$

Now in ΔPBQ,

$$\angle PBQ + \angle BPQ + \angle BQP = 180^\circ$$
$$\Rightarrow 90^\circ + 40^\circ + \angle BQP = 180^\circ$$
$$\Rightarrow \angle BQP = 50^\circ$$

In cyclic quadrilateral PQBA,

$$\angle PQB + \angle PAB = 180^\circ$$
$$\Rightarrow 50^\circ + \angle PAB = 180^\circ$$
$$\Rightarrow \angle PAB = 130^\circ$$

ii) Now in ΔPAB,

$$\angle PAB + \angle APB + \angle ABP = 180^\circ$$
$$\Rightarrow 130^\circ + \angle APB + \angle ABP = 180^\circ$$
$$\Rightarrow \angle APB + \angle ABP = 50^\circ$$

But

$$\angle APB = \angle ABP \quad (\because PA = PB)$$
$$\therefore \angle APB = \angle ABP = 25^\circ$$
$$\angle BAQ = \angle BPQ = 40^\circ$$
$$\angle APB = 25^\circ = \angle AQB \quad \text{(angles in the same segment)}$$

$$\therefore \angle AQB = 25^\circ \quad \ldots.(2)$$

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iii) Arc AQ subtends $\angle AOQ$ at the centre and $\angle APQ$ at the remaining part of the circle.

We have,

$\angle APQ = \angle APB + \angle BPQ \ldots (3)$

From (1), (2) and (3), we have

$\angle APQ = 25° + 40° = 65°$

$\therefore \angle AOQ = 2\angle APQ = 2 \times 65° = 130°$

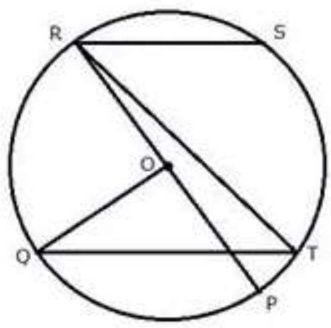
Now in ΔAOQ,

$\angle OAQ = \angle OQA \quad (\because OA = OQ)$
but
$\angle OAQ + \angle OQA + \angle AOQ = 180°$
$\Rightarrow \angle OAQ + \angle OAQ + 130° = 180°$
$\Rightarrow 2\angle OAQ = 50°$
$\Rightarrow \angle OAQ = 25°$
$\therefore \angle OAQ = \angle AQB$

But these are alternate angles.

Hence, AO is parallel to BQ.

Question 24.
The given figure shows a circle with centre O such that chord RS is parallel to chord QT, angle PRT = 20° and angle POQ = 100°.
Calculate –
i) angle QTR
ii) angle QRP
iii) angle QRS
iv) angle STR



Solution:

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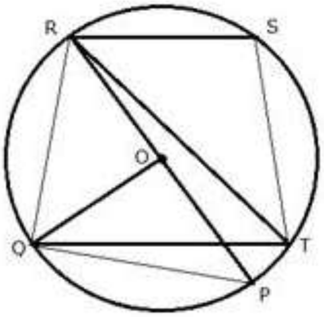

Join PQ, RQ and ST.

i)

$$\angle POQ + \angle QOR = 180^\circ$$
$$\Rightarrow 100^\circ + \angle QOR = 180^\circ$$
$$\Rightarrow \angle QOR = 80^\circ$$

Arc RQ subtends $\angle QOR$ at the centre and $\angle$ QTR at the remaining part of the circle.

$$\therefore \angle QTR = \frac{1}{2}\angle QOR$$
$$\Rightarrow \angle QTR = \frac{1}{2} \times 80^\circ = 40^\circ$$

ii) Arc QP subtends $\angle QOP$ at the centre and $\angle$ QRP at the remaining part of the circle.

$$\therefore \angle QRP = \frac{1}{2}\angle QOP$$
$$\Rightarrow \angle QRP = \frac{1}{2} \times 100^\circ = 50^\circ$$

iii) RS || QT

$$\therefore \angle SRT = \angle QTR \quad \text{(alternate angles)}$$
$$\text{but } \angle QTR = 40^\circ$$
$$\therefore \angle SRT = 40^\circ$$
Now,
$$\angle QRS = \angle QRP + \angle PRT + \angle SRT$$
$$\Rightarrow \angle QRS = 50^\circ + 20^\circ + 40^\circ = 110^\circ$$

iv) Since RSTQ is a cyclic quadrilateral

$$\therefore \angle QRS + \angle QTS = 180^\circ \text{ (sum of opposite angles)}$$

$$\Rightarrow \angle QRS + \angle QTR + \angle STR = 180^\circ$$
$$\Rightarrow 110^\circ + 40^\circ + \angle STR = 180^\circ$$
$$\Rightarrow \angle STR = 30^\circ$$

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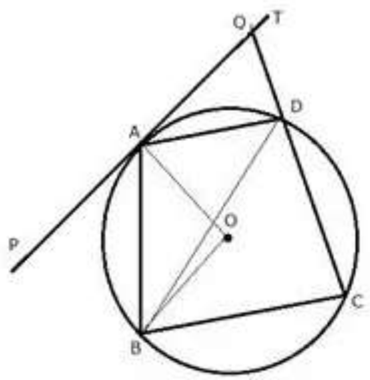
Question 25.
In the given figure, PAT is tangent to the circle with centre O, at point A on its circumference and is parallel to chord BC. If CDQ is a line segment, show that:
i) ∠BAP = ∠ADQ
ii) ∠AOB = 2∠ADQ
(iii) ∠ADQ = ∠ADB.



Solution:

i) Since PAT||BC

$\therefore \angle PAB = \angle ABC$ (alternate angles) ....(i)

In cyclic quadrilateral ABCD,

$Ext.\angle ADQ = \angle ABC$.......(ii)

from (i) and (ii)

$\angle PAB = \angle ADQ$

ii) Arc AB subtends $\angle AOB$ at the centre and $\angle ADB$ at the remaining part of the circle.

$\therefore \angle AOB = 2\angle ADB$
$\Rightarrow \angle AOB = 2\angle PAB$ (angles in alternate segments)
$\Rightarrow \angle AOB = 2\angle ADQ$ (proved in (i) part)

iii)

$\because \angle BAP = \angle ADB$ (angles in alternate segments)
but
$\angle BAP = \angle ADQ$ (proved in (i) part)
$\therefore \angle ADQ = \angle ADB$

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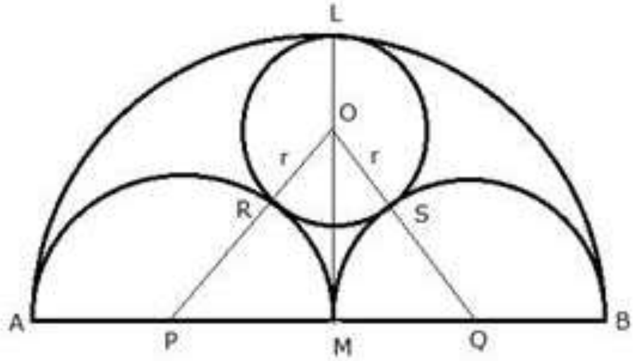
Question 26.
AB is a line segment and M is its midpoint. Three semicircles are drawn with AM, MB and AB as diameters on the same side of the line AB. A circle with radius r unit is drawn so that it touches all the three semicircles. Show that: AB = 6 x r

Solution:



Let O, P and Q be the centers of the circle and semicircles.

Join OP and OQ.

OR = OS = r

and AP = PM = MQ = QB = $\frac{AB}{4}$

Now, OP = OR + RP = r + $\frac{AB}{4}$ (since PM=RP=radii of same circle)

Similarly, OQ = OS + SQ = r + $\frac{AB}{4}$

OM = LM -; OL = $\frac{AB}{2}$ - r

Now in Rt. ΔOPM,

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$OP^2 = PM^2 + OM^2$

$$\Rightarrow \left(r + \frac{AB}{4}\right)^2 = \left(\frac{AB}{4}\right)^2 + \left(\frac{AB}{2} - r\right)^2$$

$$\Rightarrow r^2 + \frac{AB^2}{16} + \frac{rAB}{2} = \frac{AB^2}{16} + \frac{AB^2}{4} + r^2 - rAB$$

$$\Rightarrow \frac{rAB}{2} = \frac{AB^2}{4} - rAB$$

$$\Rightarrow \frac{AB^2}{4} = \frac{rAB}{2} + rAB$$

$$\Rightarrow \frac{AB^2}{4} = \frac{3rAB}{2}$$

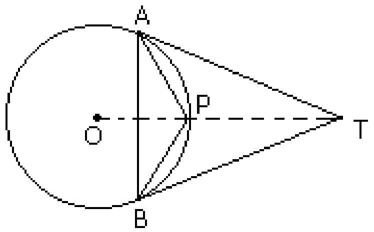
$$\Rightarrow \frac{AB}{4} = \frac{3}{2}r$$

$$\Rightarrow AB = \frac{3}{2}r \times 4 = 6r$$

Hence AB = 6 x r

Question 27.
TA and TB are tangents to a circle with centre O from an external point T. OT intersects the circle at point P. Prove that AP bisects the angle TAB.

Solution:



Join PB.

In Δ TAP and Δ TBP,

TA = TB (tangents segments from an external points are equal in length)

Also, $\angle$ ATP = $\angle$ BTP. (since OT is equally inclined with TA and TB) TP = TP (common)

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$\Rightarrow \triangle TAP \cong \triangle TBP$ (by SAS criterion of congruency)

$\Rightarrow \angle TAP = \angle TBP$ (corresponding parts of congruent triangles are equal)

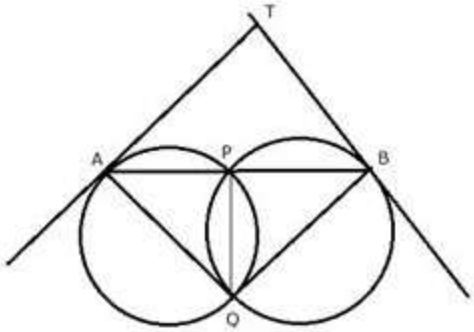
But $\angle TBP = \angle BAP$ (angles in alternate segments)

Therefore, $\angle TAP = \angle BAP$.

Hence, AP bisects $\angle TAB$.

Question 28.
Two circles intersect in points P and Q. A secant passing through P intersects the circle in A and B respectively. Tangents to the circles at A and B intersect at T. Prove that A, Q, B and T lie on a circle.

Solution:



Join PQ.

AT is tangent and AP is a chord.

$\therefore \angle TAP = \angle AQP$ (angles in alternate segments) ....(i)

Similarly, $\angle TBP = \angle BQP$ ....(ii)

Adding (i) and (ii)

$\angle TAP + \angle TBP = \angle AQP + \angle BQP$
$\Rightarrow \angle TAP + \angle TBP = \angle AQB$............(iii)

Now in $\triangle TAB$,

$\angle ATB + \angle TAP + \angle TBP = 180^\circ$
$\Rightarrow \angle ATB + \angle AQB = 180^\circ$

Therefore, AQBT is a cyclic quadrilateral.

Hence, A, Q, B and T lie on a circle.

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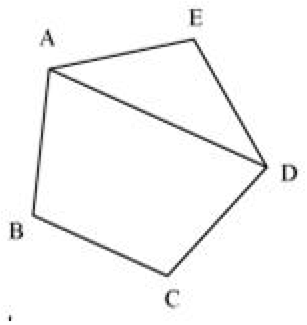
Question 29.
Prove that any four vertices of a regular pentagon are concyclic (lie on the same circle)

Solution:



ABCDE is a regular pentagon.

$\therefore \angle BAE = \angle ABC = \angle BCD = \angle CDE = \angle DEA = \left(\frac{5-2}{5}\right) \times 180^\circ = 108^\circ$

In $\triangle$ AED,

AE = ED (Sides of regular pentagon ABCDE)

$\therefore \angle EAD = \angle EDA$

In $\triangle$ AED,

$\angle AED + \angle EAD + \angle EDA = 180^\circ$

$\Rightarrow 108^\circ + \angle EAD + \angle EAD = 180^\circ$

$\Rightarrow 2\angle EAD = 180^\circ - 108^\circ = 72^\circ$

$\Rightarrow \angle EAD = 36^\circ$

$\therefore \angle EDA = 36^\circ$

$\angle BAD = \angle BAE - \angle EAD = 108^\circ - 36^\circ = 72^\circ$

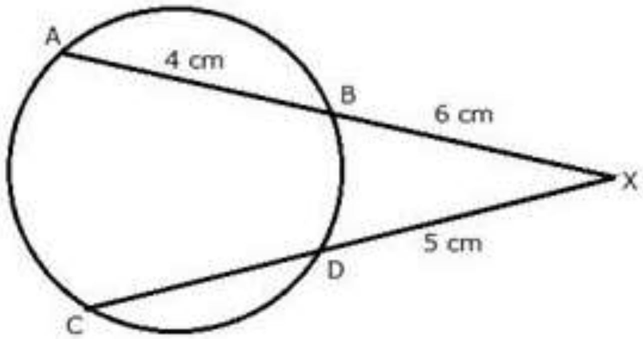
In quadrilateral ABCD,

$\angle BAD + \angle BCD = 108^\circ + 72^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral

Question 30.
Chords AB and CD of a circle when extended meet at point X. Given AB = 4 cm, BX = 6 cm and XD = 5 cm. Calculate the length of CD.

Solution:



We know that XB.XA = XD.XC

Or, XB.(XB + BA) = XD.(XD + CD)

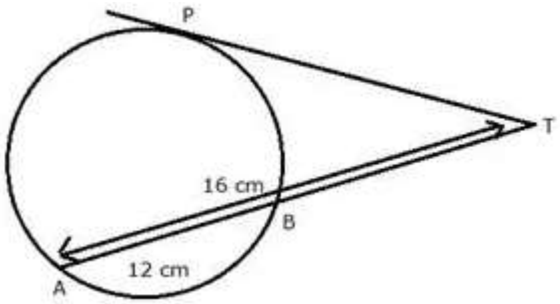
Or, 6(6 + 4) = 5(5 + CD)

Or, 60 = 5(5 + CD)

Or, $5 + CD = \frac{60}{5} = 12$

Or, CD = 12 - 5 = 7 cm.

Question 31.
In the given figure, find TP if AT = 16 cm and AB = 12 cm.



Solution:

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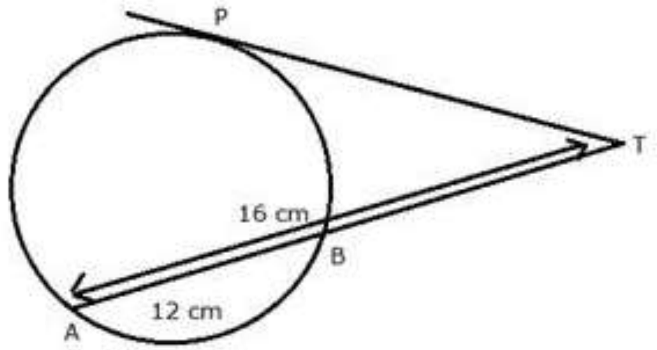


PT is the tangent and TBA is the secant of the circle.

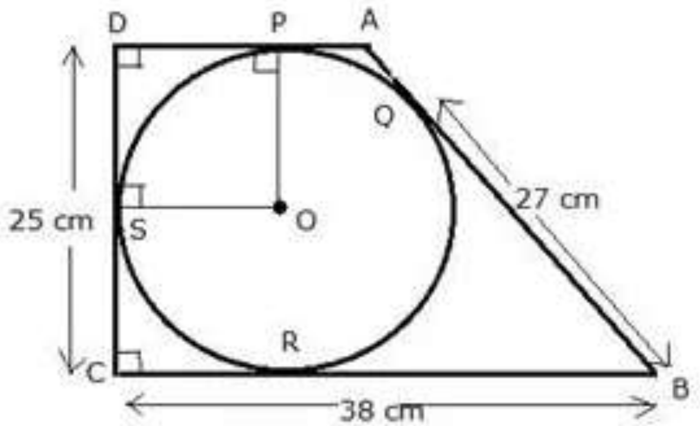
Therefore, $TP^2 = TA \times TB$

$TP^2 = 16 \times (16\text{-}12) = 16 \times 4 = 64 = (8)^2$

Therefore, TP = 8 cm

Question 32.

In the following figure, A circle is inscribed in the quadrilateral ABCD.



If BC = 38 cm, QB = 27 cm, DC = 25 cm and that AD is perpendicular to DC, find the radius of the circle.

Solution:

From the figure we see that BQ = BR = 27 cm (since length of the tangent segments from an external point are equal)

As BC = 38 cm

$\Rightarrow$CR = CB - BR = 38 - 27

= 11 cm

Again,

CR = CS = 11cm (length of tangent segments from an external point are equal)

Now, as DC = 25 cm

$\therefore$ DS = DC - SC

= 25 -11

= 14 cm

Now, in quadrilateral DSOP,

$\angle$PDS = 90° (given)

$\angle$OSD = 90°, $\angle$OPD = 90° (since tangent is perpendicular to the radius through the point of contact)

$\Rightarrow$DSOP is a parallelogram

$\Rightarrow$OP||SD and $\Rightarrow$PD||OS

Now, as OP = OS (radii of the same circle)

$\Rightarrow$OPDS is a square. $\therefore$ DS = OP = 14cm

$\therefore$ radius of the circle = 14 cm

Question 33.
In the figure, XY is the diameter of the circle, PQ is the tangent to the circle at Y. Given that ∠AXB = 50° and ∠ABX = 70°. Calculate ∠BAY and ∠APY.

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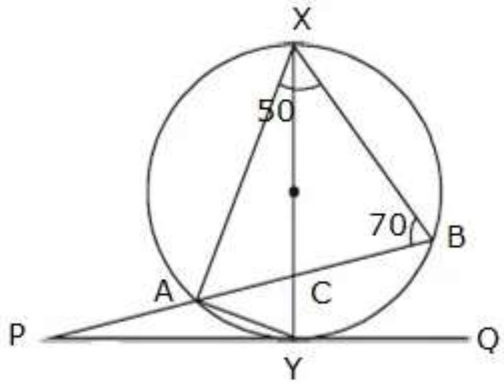
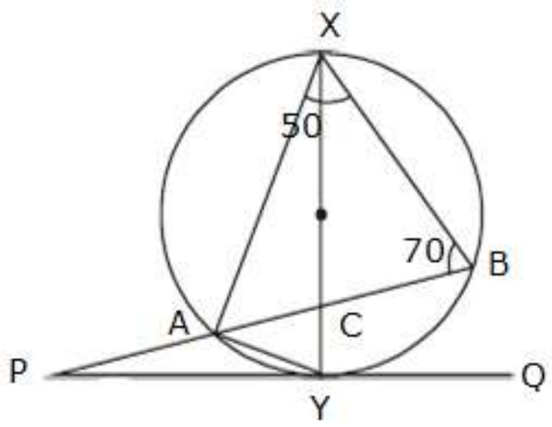


Solution:



In Δ AXB,

$\angle XAB + \angle AXB + \angle ABX = 180^\circ$ [Triangle property]

$\Rightarrow \angle XAB + 50^\circ + 70^\circ = 180^\circ$

$\Rightarrow \angle XAB = 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow \angle XAY = 90^\circ$ [Angle of semi-circle]

$\therefore \angle BAY = \angle XAY - \angle XAB = 90^\circ - 60^\circ = 30^\circ$

and $\angle BXY = \angle BAY = 30^\circ$ [Angle of same segment]

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$\therefore\ \angle ACX = \angle BXY + \angle ABX$ [External angle = Sum of two interior angles]

$= 30° + 70°$

$= 100°$

also,

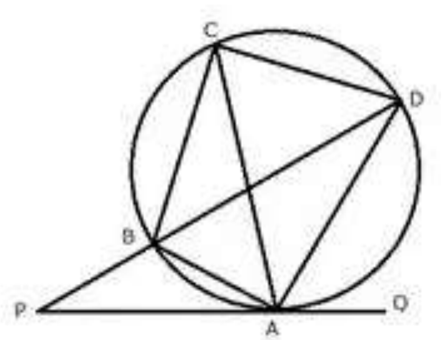
$\angle XYP = 90°$ [Diameter ⊥ tangent]

$\angle APY = \angle ACX - \angle CYP$

$\angle APY = 100° - 90°$

$\angle APY = 10°$

Question 34.
In the given figure, QAP is the tangent at point A and PBD is a straight line. If ∠ACB = 36° and ∠APB = 42°; find:
i) ∠BAP
ii) ∠ABD
iii) ∠QAD
iv) ∠BCD



Solution:

PAQ is a tangent and AB is a chord of the circle.

i) $\therefore\ \angle BAP = \angle ACB = 36°$ (angles in alternate segment)

ii) In ΔAPB,

$Ext.\angle ABD = \angle APB + \angle BAP$
$\Rightarrow Ext\angle ABD = 42° + 36° = 78°$

iii) $\angle ADB = \angle ACB = 36°$ (angles in the same segment)

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Now in ΔPAD,

$$\text{Ext}\angle QAD = \angle APB + \angle ADB$$
$$\Rightarrow \text{Ext}\angle QAD = 42^\circ + 36^\circ = 78^\circ$$

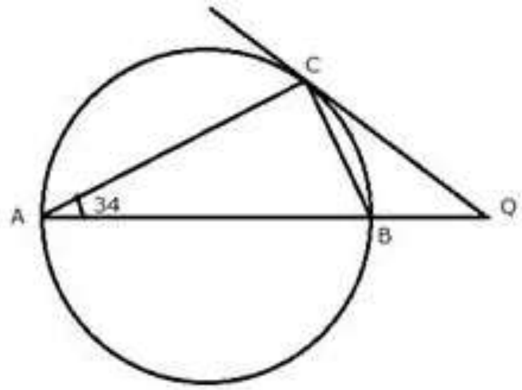
iv) PAQ is the tangent and AD is chord

$\therefore \angle QAD = \angle ACD = 78^\circ$ (angles in alternate segment)

and $\angle BCD = \angle ACB + \angle ACD$

$\therefore \angle BCD = 36^\circ + 78^\circ = 114^\circ$

Question 35.
In the given figure, AB is the diameter. The tangent at C meets AB produced at Q.



If
$\angle CAB = 34°$, find
i) $\angle CBA$
ii) $\angle CQB$

Solution:

i) AB is diameter of circle.

$\therefore ACB = 90^\circ$

In ΔABC,

$$\angle A + \angle B + \angle C = 180^\circ$$
$$\Rightarrow 34^\circ + \angle CBA + 90^\circ = 180^\circ$$
$$\Rightarrow \angle CBA = 56^\circ$$

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ii) QC is tangent to the circle

$\therefore \angle CAB = \angle QCB$

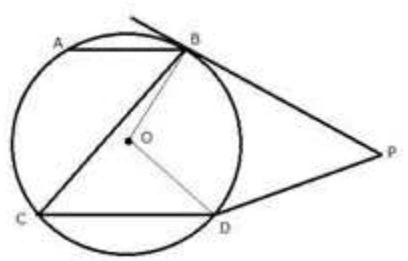
Angle between tangent and chord = angle in alternate segment

$\therefore \angle QCB = 34°$

ABQ is a straight line

$\Rightarrow \angle ABC + \angle CBQ = 180°$
$\Rightarrow 56° + \angle CBQ = 180°$
$\Rightarrow \angle CBQ = 124°$
Now,
$\angle CQB = 180° - \angle QCB - \angle CBQ$
$\Rightarrow \angle CQB = 180° - 34° - 124°$
$\Rightarrow \angle CQB = 22°$

Question 36.
In the given figure, O is the centre of the circle. The tangents at B and D intersect each other at point P.



If AB is parallel to CD and $\angle ABC = 55°$, find:
i) $\angle BOD$
ii) $\angle BPD$

Solution:
i)

$\angle BOD = 2\angle BCD$
$\Rightarrow \angle BOD = 2 \times 55° = 110°$

ii) Since, BPDO is cyclic quadrilateral, opposite angles are supplementary.

$\therefore \angle BOD + \angle BPD = 180°$
$\Rightarrow \angle BPD = 180° - 110° = 70°$

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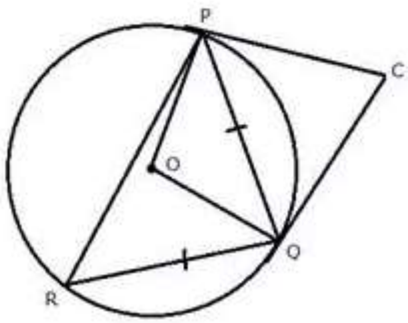
Question 37.
In the figure given below PQ =QR, ∠RQP = 68°, PC and CQ are tangents to the circle with centre O. Calculate the values of:
i) ∠QOP
ii) ∠QCP



Solution:
i) PQ = RQ

∴ ∠PRQ = ∠QPR (opposite angles of equal sides of a triangle)

$\Rightarrow \angle PRQ + \angle QPR + 68° = 180°$
$\Rightarrow 2\angle PRQ = 180° - 68°$
$\Rightarrow \angle PRQ = \frac{112°}{2} = 56°$

Now, ∠ QOP = 2∠ PRQ (angle at the centre is double)

$\Rightarrow \angle QOP = 2 \times 56° = 112°$

ii) ∠ PQC = ∠ PRQ (angles in alternate segments are equal)

∠ QPC = ∠ PRQ (angles in alternate segments)

$\therefore \angle PQC = \angle QPC = 56° \quad (\because \angle PRQ = 56° \text{ from(i)})$
$\angle PQC + \angle QPC + \angle QCP = 180°$
$\Rightarrow 56° + 56° + \angle QCP = 180°$
$\Rightarrow \angle QCP = 68°$

Question 38.
In two concentric circles prove that all chords of the outer circle, which touch the inner circle, are of equal length.

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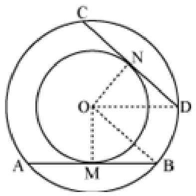
Solution:

Consider two concentric circles with centres at O. Let AB and CD be two chords of the outer circle which touch the inner circle at the points M and N respectively.



To prove the given question, it is sufficient to prove AB = CD.

For this join OM, ON, OB and OD.

Let the radius of outer and inner circles be R and r respectively.

AB touches the inner circle at M.

$\therefore$ AB is a tangent to the inner circle

$\therefore OM \perp AB$

$\Rightarrow BM = \frac{1}{2}AB$

$\Rightarrow AB = 2BM$

Similarly $ON \perp CD$, and CD = 2DN

Using Pythagoras theorem in Δ OMB and Δ OND

$OB^2 = OM^2 + BM^2, \quad OD^2 = ON^2 + DM^2$

$\Rightarrow BM = \sqrt{R^2 - r^2}, \quad DN = \sqrt{R^2 - r^2}$

Now,

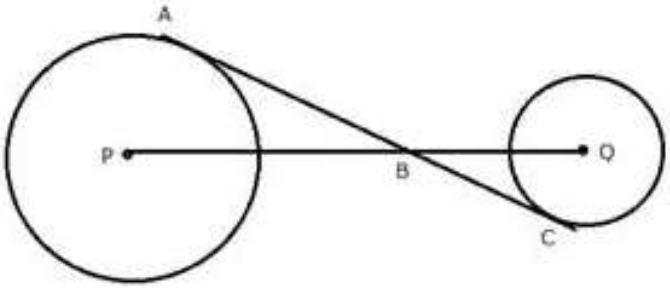
$AB = 2BM = 2\sqrt{R^2 - r^2}, \quad CD = 2DN = 2\sqrt{R^2 - r^2}$

$\therefore AB = CD$

Hence Proved.

Question 39.

In the figure, given below, AC is a transverse common tangent to two circles with centers P and Q and of radii 6 cm and 3 cm respectively.
Given that AB = 8 cm, calculate PQ.



Solution:

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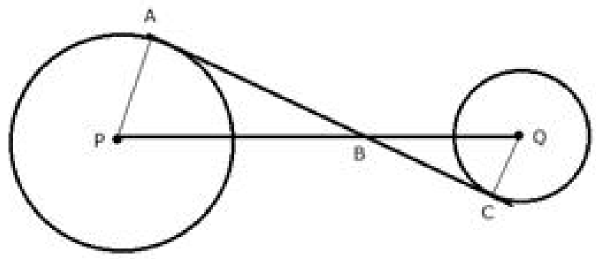

Since AC is tangent to the circle with center P at point A.

$\therefore \angle PAB = 90^\circ$

Similarly, $\angle QCB = 90^\circ$

In ΔPAB and ΔQCB,

$\angle PAB = \angle OCB = 90^\circ$

$\angle PBA = \angle QBC$ (vertically opposite angles)

$\therefore \Delta PAB \sim \Delta QCB$

$$\Rightarrow \frac{PA}{QC} = \frac{PB}{QB} \text{.............(i)}$$

Also in Rt. ΔPAB,

$$PB = \sqrt{PA^2 + PB^2}$$

$$\Rightarrow PB = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm.......(ii)}$$

From (i) and (ii),

$$\frac{6}{3} = \frac{10}{QB}$$

$$\Rightarrow QB = \frac{3 \times 10}{6} = 5 \text{ cm}$$

Now,

$PQ = PB + QB = (10 + 5)$ cm $= 15$ cm

Question 40.
In the figure given below, O is the centre of the circum circle of triangle XYZ. Tangents at X and Y intersect at point T. Given ∠XTY = 80° and ∠XOZ = 140°, calculate the value of ∠ZXY.

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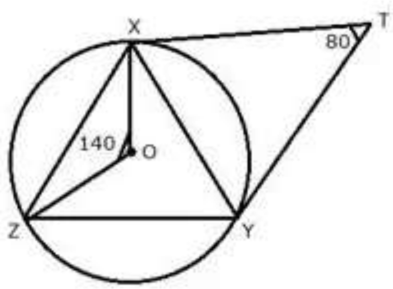
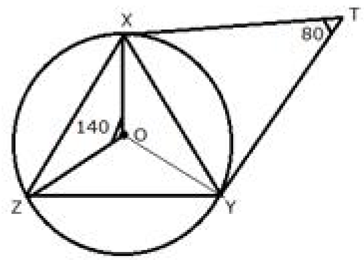


Solution:



In the figure, a circle with centre O, is the circum circle of triangle XYZ.

$\angle XOZ = 140°$

Tangents at X and Y intersect at point T, such that $\angle XTY = 80°$

$\therefore \angle XOY = 180° - 80° = 100°$

But, $\angle XOY + \angle YOZ + \angle ZOX = 360°$ [Angles at a point]

$\Rightarrow 100° + \angle YOZ + 140° = 360°$

$\Rightarrow 240° + \angle YOZ = 360°$

$\Rightarrow \angle YOZ = 360° - 240°$

$\Rightarrow \angle YOZ = 120°$

Now arc YZ subtends $\angle YOZ$ at the centre and $\angle YXZ$ at the remaining part of the circle

$\therefore \angle YOZ = 2\angle YXZ$

$\Rightarrow \angle YXZ = \frac{1}{2}\angle YOZ$

$\Rightarrow \angle YXZ = \frac{1}{2} \times 120° = 60°$

Question 41.

In the given figure, AE and BC intersect each other at point D. If ∠CDE=90°, AB = 5 cm, BD = 4 cm and CD = 9 cm, find AE.

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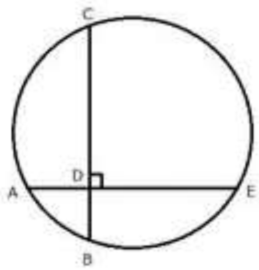
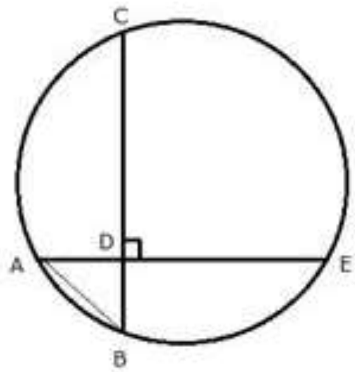


Solution:



From Rt. ΔADB,

$$AD = \sqrt{AB^2 - DB^2} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3 \text{ cm}$$

Now, since the two chords AE and BC intersect at D,

$AD \times DE = CD \times DB$

$3 \times DE = 9 \times 4$

$$DE = \frac{9 \times 4}{3} = 12$$

Hence, $AE = AD + DE = (3 + 12) = 15$ cm

Question 42.
In the given circle with centre O, ∠ABC = 100°, ∠ACD = 40° and CT is a tangent to the circle at C. Find ∠ADC and ∠DCT.

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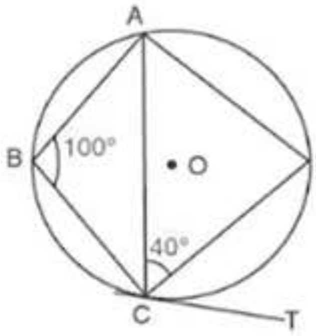


Solution:

In a cyclic quadrilateral ABCD,

$\angle ABC + \angle ADC = 180^\circ$ ....(opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow 100^\circ + \angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 80^\circ$

Now, in ΔACD,

$\angle ACD + \angle CAD + \angle ADC = 180^\circ$

$\Rightarrow 40^\circ + \angle CAD + 80^\circ = 180^\circ$

$\Rightarrow \angle CAD = 180^\circ - 120^\circ$

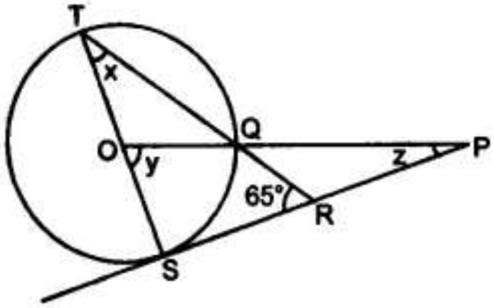
$\Rightarrow \angle CAD = 60^\circ$

Now $\angle DCT = \angle CAD$ ....(angles in the alternate segment are equal)

$\therefore \angle DCT = 60^\circ$

Question 43.

In the figure given below, O is the centre of the circle and SP is a tangent. If $\angle SRT = 65^\circ$, find the values of x, y and z.



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Solution:

$TS \perp SP,$
$\Rightarrow \angle TSR = 90^\circ$
In $\Delta TSR,$
$\angle TSR + \angle TRS + \angle RTS = 180^\circ$
$\Rightarrow 90^\circ + 65^\circ + x = 180^\circ$
$\Rightarrow x = 180^\circ - 90^\circ - 65^\circ$
$\Rightarrow x = 25^\circ$

Now, $y = 2x$ .... (Angle subtended at the centre is double that of the angle subtended by the arc at the same centre)

$\Rightarrow y = 2 \times 25^\circ$
$\Rightarrow y = 50^\circ$

In $\Delta OSP,$
$\angle OSP + \angle SPO + \angle POS = 180^\circ$
$\Rightarrow 90^\circ + z + 50^\circ = 180^\circ$
$\Rightarrow z = 180^\circ - 140^\circ$
$\Rightarrow z = 40^\circ$

Hence, $x = 25^\circ$, $y = 50^\circ$ and $z = 40^\circ$

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